
TEMPORAL FLUCTUATIONS IN BIORHYTHMS: EXPRESSION OF SELF-ORGANIZED CRITICALITY?

KLAUS-D. KNIFFKI, WOLFGANG MANDEL* and
PHUOC TRAN-GIA*

Physiologisches Institut and Institut für Informatik,
Röntgenring 9 and Am Hubland, Universität Würzburg,
D-97070 Würzburg, Germany*

Abstract

Recently, a general organizing principle has been reported connecting $1/f$ -noise with the self-similar scale-invariant 'fractal' properties in space, hence reflecting two sides of a coin, the so-called *self-organized critical state*. The basic idea is that dynamical systems with many degrees of freedom operate persistently far from equilibrium at or near a threshold of stability at the border of chaos. Temporal fluctuations which cannot be explained as consequences of statistically independent random events are found in a variety of physical and biological phenomena. The fluctuations of these systems can be characterized by a power spectrum density $S(f)$ decaying as f^{-b} at low frequencies with an exponent $b < 1.5$. We present a new approach to describe the individual biorhythm of humans using data from a colleague who has kept daily records for two years of his state of well-being applying a fifty-point magnitude category scale. This time series was described as a point process by introducing two discriminating rating levels R for the occurrence of $R \geq 40$ and $R \leq 10$.

For $b < 1$ a new method to estimate the low frequency part of $S(f)$ was applied using counting statistics without applying Fast Fourier Transform. The method applied reliably discriminates these types of fluctuations from a random point process, with $b = 0.0$. It is very tempting to speculate that the neural mechanisms at various levels of the nervous system underlying the perception of different values of the subjective state of well-being, are expressions of a self-organized critical state.

1. INTRODUCTION

A variety of phenomena in nature exhibit temporal fluctuations in the absence of intentional stimulation which cannot be explained as a consequence of statistically independent random events. It has been shown that temporal fluctuations found in phenomena as different as membrane currents, earthquakes, sunspot activity, light emitted from quasars, sand falling through an hour glass, traffic flow, heart beat or breathing activity, can be characterized by their power spectrum density $S(f)$ decaying as f^{-b} at low frequencies with $0.5 \leq b \leq 1.5$. This behavior of the temporal fluctuations of a system described by its $S(f)$ is called $1/f$ -noise.

Recently, it has been suggested¹ that the large fluctuations in time characterized as $1/f$ -fluctuations and the self-similarity in space might both be manifestations of a self-organized critical state. Self-organized criticality (SOC) describes the tendency of some open dissipative many-body systems to drive themselves spontaneously to a critical state with no characteristic time or length scales without any fine-tuning by external fields: hence the criticality is self-organized. This is in contrast to the criticality of equilibrium systems undergoing phase transition only at a critical external field, such as temperature, pressure, electrical or magnetic field. The idea provides a unifying concept for large scale behavior in systems with many degrees of freedom operating persistently far from equilibrium at or near a threshold of instability, so to speak on the "border to chaos".²

The SOC phenomenon is expected to be quite universal and we assume that it is the underlying principle of some biological many-body systems. We present a new approach to describe the individual biorhythm in humans using data from a colleague who has kept standardized daily records for two years of his state of general well-being applying a fifty-point magnitude category scale and analyze the temporal fluctuations by estimating the power spectrum density in its low frequency range to characterize the self-similar temporal rating sequences.

2. METHODS

The subjective intensity of well-being was measured with a linear category scaling procedure (category partitioning)³ with five categories each subdivided in ten steps: 1–10: very strong "down", 11–20: strong "down", 21–30: moderate "down"/moderate "up", 31–40: strong "up", 41–50: very strong "up". Thus, the subject could, after choosing a major category, fine-tune the rating of well-being by choosing a number within that main category. In general, the daily ratings were performed at 6.00 a.m. and stored for subsequent analysis. Occasionally, fluctuations within a day of the subjective well-being were observed, but were not monitored and therefore neglected in this analysis.

As shown in Fig. 1, the time series of the daily ratings R can be described as a point process by introducing discriminating rating levels for the occurrence of $R \geq r$, e.g. for the occurrence of "ups" (cf. Fig. 2) and $R \leq s$, e.g. for the occurrence of "downs".

Usually $S(f)$ is obtained by Fast Fourier Transform (FFT). To avoid the well-known problems in using FFT for the obtained point process, we used a new simple method based on counting statistics⁴ to analyse the low frequency part of $S(f)$ of the recorded ratings of human general well-being.

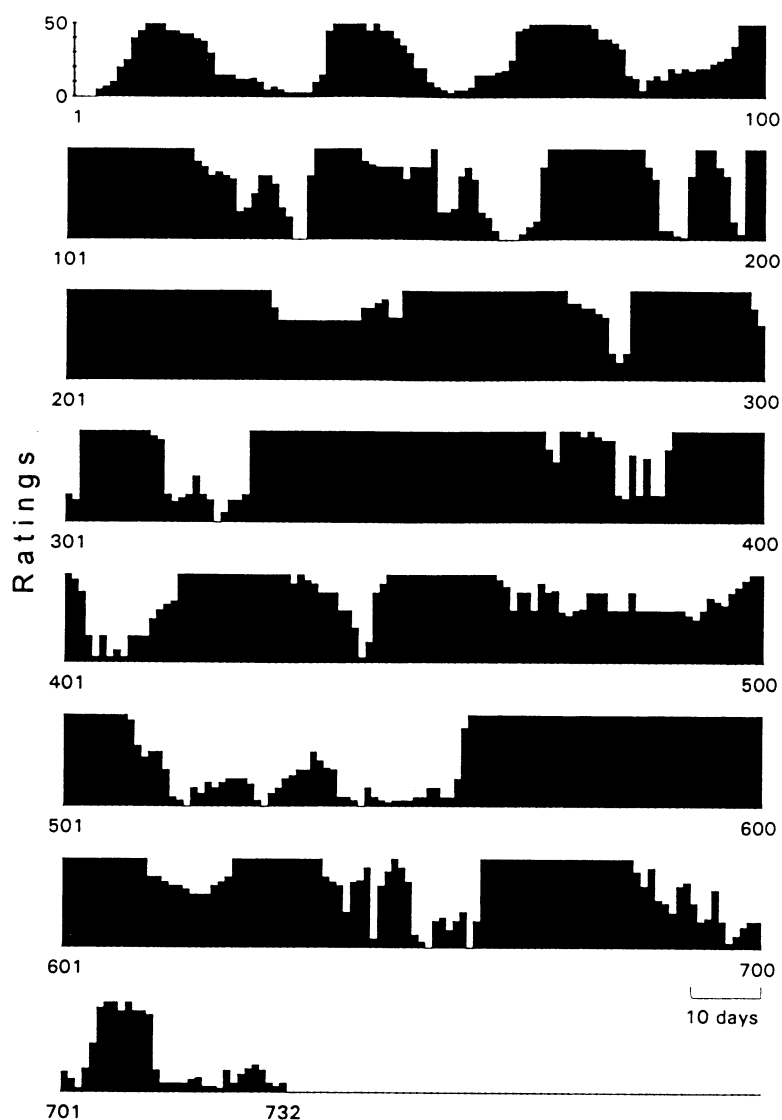


Fig. 1 Daily ratings of the subjective well-being (biorhythm) for two years using a linear category scaling procedure.

The series of recorded ratings after introducing a discriminating rating level is considered to be a point process described as

$$y(t) = \sum_{i=1}^n \delta(t - t_i) \quad (1)$$

in which $\delta(t - t_i)$ represents Dirac's delta function and t_i is the time of occurrence of a particular $R \geq r$ or $R \leq s$ within the train of n events. In the absence of severe intentional stimulation $y(t)$ is assumed to be statistically stationary. Another statistical variable derived from Eq. (1) is the actual number of events $N(\Delta t)$ occurring in a time interval Δt ranging from t_1 to t_2 . Thus, $N(\Delta t)$ can be expressed as

$$N(\Delta t) = \int_{t_1}^{t_2} \sum_j^{\nu} \delta(t - t_j) dt. \quad (2)$$

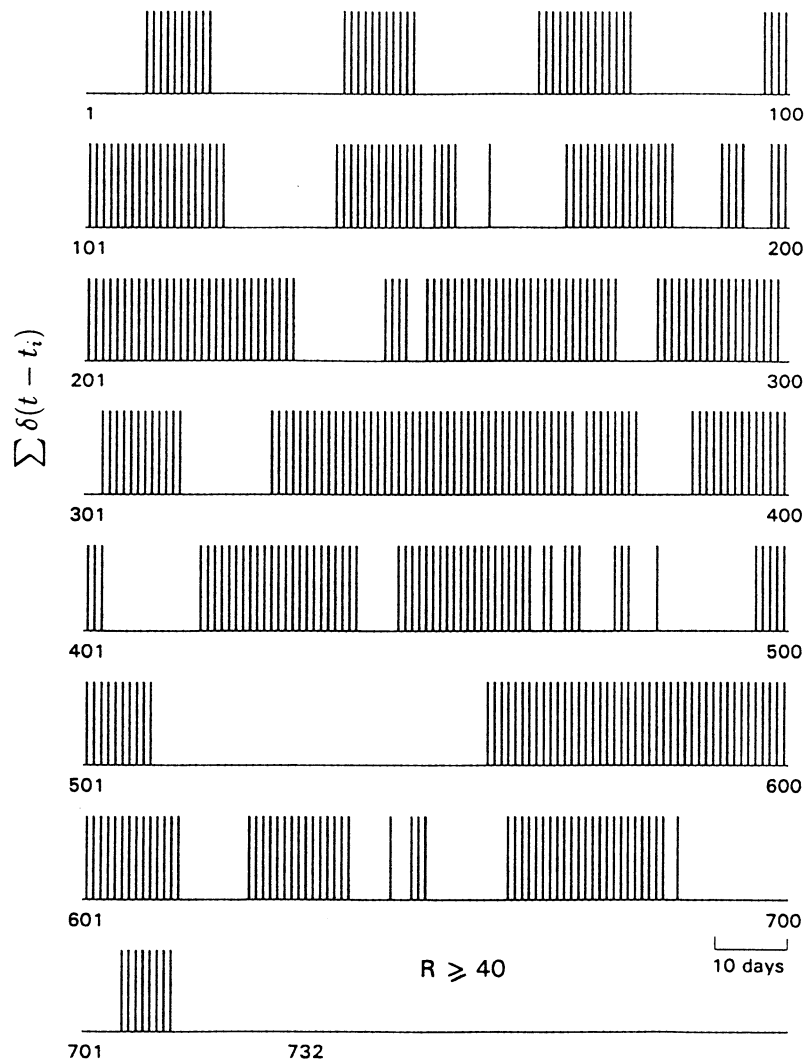


Fig. 2 Occurrence of ratings of the subjective well-being with $R \geq 40$ of the entire data set shown in Fig. 1. The corresponding days are marked by Dirac's delta functions $\delta(t - t_i)$.

The second time derivative of the variance of counts $Var[N(\Delta t)]$, the so-called variance-time curve, is related to the auto-covariance function of y , $C_y(\Delta t)$ by⁵

$$C_y(\Delta t) = \frac{1}{2} (Var[N(\Delta t)])'' \tag{3}$$

and therefore the key to determine the low frequency part of the spectrum $S_y(f)$ is to experimentally obtain $Var[N(\Delta t)]$.⁴ If the variance-time curve follows within certain limits a power law

$$Var[N(\Delta t)] \sim (\Delta t)^{1+b} \quad \text{with } b < 1, \tag{4}$$

then it can be shown using the Wiener-Chinchin theorem that the spectrum $S_y(f)$ scales as

$$S_y(f) \sim f^{-b} \tag{5}$$

within $f_{\min} < f < f_{\max}$.^{4,6}

The variance-time curve is defined by the variance of counts for time intervals of length Δt as

$$\text{Var}[N(\Delta t)] = \langle N^2(\Delta t) \rangle - \langle N(\Delta t) \rangle^2 \quad (6)$$

with $\langle \dots \rangle$ denoting expectation values. For estimating $\text{Var}[N(\Delta t)]$, the entire observation time T is divided into k counting windows of duration Δt with $T = k\Delta t$ and the variance of counts is determined for this particular window Δt . This is repeated for different values of Δt . The results were plotted as $\text{Var}[N(\Delta t)]$ vs Δt on a log-log scale and fitted by linear regression using the least square method.

3. RESULTS

In Fig. 1 the whole data set is shown, i.e., the daily ratings for two years are displayed. It is obvious from the data, that the state of subjective well-being is not constant but fluctuates in general from day to day. By no means these fluctuations taken as a whole are simple oscillations describable by a sine function as it is often assumed by performing the so-called biorhythm analysis.⁷ In a rough approximation the data look as if the basic underlying mechanism responsible for the subjective well-being is a two-state (“up”–“down”) system with a certain endogenic dynamics.

By introducing discriminating rating levels for the occurrence of $R \geq r$ to reveal the fluctuations in the “up”, the data shown in Fig. 1 were transformed into a point process. To obtain Fig. 2 the discriminating rating level was set to $R \geq 40$, i.e. the point process shows the fluctuations of the strong “ups” of the subjective well-being irrespective of their actual rating. Similar point processes showing the clusters of events described as $\delta(t - t_i)$ for other discriminating rating levels were obtained and analyzed. In particular, for determining the fluctuations in the occurrence of the “downs” $R \leq 10$ were chosen.

After introducing a certain $R \geq r / \leq s$, for the resulting point process the low frequency part of the spectrum $S(f)$ was determined by using counting statistics as described in Methods. Fig. 3 shows the result of the point process shown in Fig. 2, i.e. the $\text{Var}[N(\Delta t)]$ is plotted on a log-log scale versus the counting windows Δt . From the straight lines fitted to the data points it is demonstrated that the variance-time curve follows the power law

$$\text{Var}[N(\Delta t)] \sim (\Delta t)^{1+0.65} \quad (7)$$

for the scaling region $1d \leq \Delta t \leq 15d$ and thus the low frequency part of the spectrum scales as

$$S(f) \sim f^{-0.65} \quad (8)$$

For $\Delta t > 15d$ a second scaling region was observed showing an almost random behavior ($b = 0.17$).

Similar results, i.e. similar scaling behavior for the variance-time curve and for the spectrum was obtained for other discriminating levels, in particular for $R \leq 10$, i.e. for the “downs”.

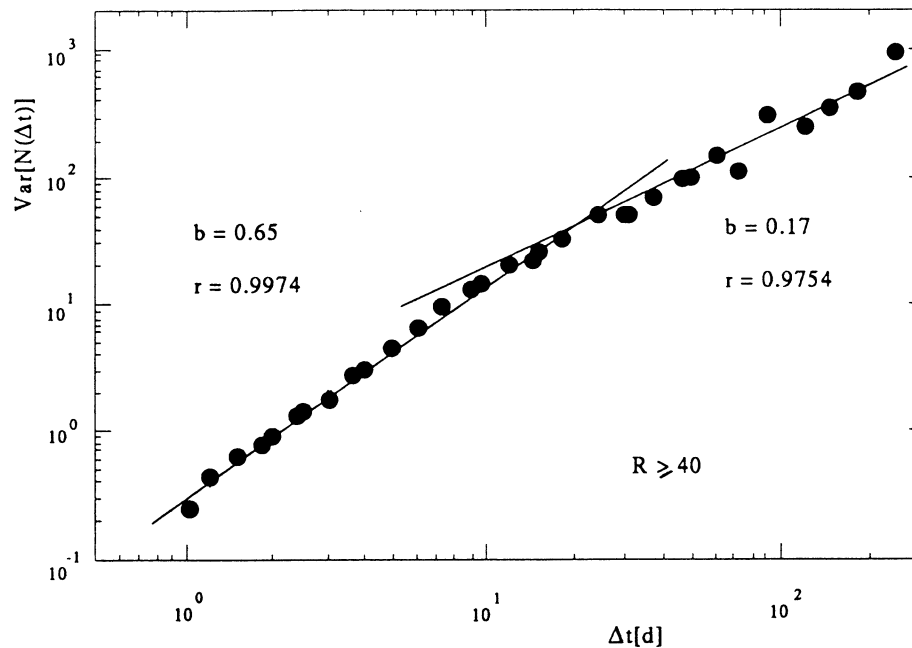


Fig. 3 The variance-time curve $\text{Var}[N(\Delta t)]$ for the point process shown in Fig. 2 plotted on a log-log scale versus the counting windows Δt . The variance-time curve scales as $(\Delta t)^{1+b}$ for $1d \leq \Delta t \leq 15d$ with $b = 0.65$ obtained by linear regression using the least square method with the indicated correlation coefficient r . For the second scaling region, i.e. for $\Delta t \geq 15d$ the exponent $b = 0.17$ indicates more random fluctuations of the ratings $R \geq 40$.

4. DISCUSSION

Recently, also due to the introduction of the concept of self-organized criticality,¹ attention has been drawn to the characterization of temporal fluctuations in a number of physical and biological systems. In the following the discussion will be focused on the fluctuations of endogenous biological rhythms.

The human heart rate, even in the healthy resting subject, displays a considerable amount of fluctuations, which have been characterized as $1/f$ -fluctuations.⁸⁻¹¹ Furthermore, it was demonstrated that the heart rate variability of healthy men shows periods of $1/f$ -fluctuations with interpolated periods of white noise within 24 hours.^{4,13}

In animal experiments it has been demonstrated that the fluctuations in respiratory intervals also exhibited $1/f$ -fluctuations, but these characteristic types of fluctuation disappeared into white noise fluctuations when the end-tidal pCO_2 was raised to 50 or 60 mmHg.¹⁴

The fluctuating insulin requirements of an unstable diabetic over an eight-year period have been subjected to spectral analysis and it was demonstrated that the low frequency part of the spectrum did exhibit $1/f$ characteristics.¹⁵

Recently, the spectral analysis of the discharge of neurones located in the mesencephalic reticular formation during paradoxical sleep of the cat has revealed that in this state of the animal $1/f$ -fluctuations of the neuronal discharge do exist. However, the low frequency spectral profile became flat, i.e. white noise was found during slow-wave sleep.^{16,17} So far, also the thalamic neuronal discharge exhibited $1/f$ -fluctuations in the absence of intentional stimulation, but we have not seen the transition into white noise fluctuations.^{18,19} Earlier,

even for the discharges in primary afferent auditory fibres $1/f$ characteristics have been reported.²⁰

It is tempting to speculate that the basic mechanisms underlying the neuronal and humoral activity in the central nervous system responsible for the subjective state of well-being in the absence of intentional stimulation are expressions of a self-organized critical state as introduced for physical systems.¹ Self-organized criticality (SOC) describes the tendency of dissipative systems with many degrees of freedom to drive themselves to a critical state with a wide range of length and time scales without any fine-tuning of external fields. The idea complements the concept of chaos, wherein simple systems with a small number of degrees of freedom can display quite complex behavior.²¹

Currently, it is hard to give a rigorous definition for SOC; however, usually this name is given to those systems which do not need fine-tuning of external fields to give power-law characteristics for the parameters describing the system. The canonical example of SOC is the cellular automaton model called "sand-pile model".¹ The critical state is characterized by "avalanches" (activity) with power-law spatial and temporal distribution functions limited only by the size of the system. We assume that the subjective well-being dynamics can be described as a self-organized critical process and characterize the temporal fluctuations by its low frequency part of the power spectrum. The method applied reliably discriminates f^{-b} fluctuations with $b = 0.65$ in our case in the first scaling region (cf. Fig. 3) from a random point process, which would result in $b = 0.0$.

The exponent b will converge to the theoretical value only for infinite long sequences of $R\delta(t - t_i)$.⁴ Since in praxi the ratings are confined to limited time periods T_{\max} , the exponent b will be a function of T_{\max} , i.e. $b = b(T_{\max})$. In our case $T_{\max} = 2$ years, which could result in an underestimation of the exponent b (cf. Ref. 4). In any case, the time sequence of events with $R \geq 40$ (Fig. 2) is not the representation of a Poisson process, i.e., of white noise. Therefore, without external stimulation based on one's own monitored biorhythm for a given time period it should be possible to predict future episodes with a certain probability by applying modified feed-forward neural networks learning with the backpropagation algorithm.

If the neuronal/humoral system responsible for the subjective well-being is indeed operating at a critical state, an external perturbation can create either a small effect or a large one. There is in principle no limit on how long the effect may last. The degree of unpredictability is actually less severe than for chaotic systems; SOC systems are operating at the "border of chaos".² In SOC systems due to an external perturbation the maximum predictability decays as a power law, t^{-a} , where a is some constant.²² Fluctuations due to external stimulation are much stronger in SOC systems than those being realized in an equilibrium system and cannot be prevented. In case of the described biorhythm this would mean that a transition from the "up" state to the "down" state due to a severe external perturbation is inevitable for the individual.

As a very rough approximation the biorhythm displayed in Fig. 1 may be described by a two-state, i.e., an "up"–"down" system with an intrinsic dynamics. Currently, with the limited amount of data it is impossible to decide whether the neuronal/humoral system responsible for the biorhythm is a representation of a general process which has been studied under the name *stochastic resonance*^{23,24} or is the realization of an *alternating fractal renewal process*.²⁵

REFERENCES

1. P. Bak, C. Tang and K. Wiesenfeld, "Self-organized Criticality: An Explanation of $1/f$ Noise," *Phys. Rev. Lett.* **59**, 381–384 (1987).
2. P. Bak, "Is the World at the Border of Chaos?" *Ann. N. Y. Acad. Sci.* **581**, 110–118 (1990a).
3. O. Heller, "Hörfeldaudiometrie mit dem Verfahren der Kategorienunterteilung (KU)," *Psychol. Beitr.* **27**, 478–493 (1985).
4. M. Meesmann, F. Grüneis, P. Flachenecker and K.-D. Kniffki, "A New Method of Analysis of Heart Rate Variability: Counting Statistics of $1/f$ Fluctuations," *Biol. Cybern.* **68**, 299–306 (1993a).
5. D. R. Cox and V. Isham, *Point Processes* (Chapman and Hall, London 1980).
6. M. Meesmann and K.-D. Kniffki, in preparation.
7. W. A. Appel, *Biorhythmik*, Falken-Verlag, Niedernhausen, 1982.
8. M. Kobayashi and T. Musha, " $1/f$ -Fluctuations of Heartbeat Period," *IEEE Trans. Biomed. Eng. BME* **29**, 456–457 (1982).
9. J. P. Saul, P. Albrecht, R. D. Berger and R. J. Cohen, "Analysis of Long Term Heart Rate Variability: Methods, $1/f$ -Scaling and Implications," *Comput. Cardiol.* **14**, 419–422 (1987).
10. J. P. Zbilut, G. Mayer-Kress, P. A. Sobotka, M. O'Toole and J. X. Thomas Jr., "Bifurcations and Intrinsic Chaotic and $1/f$ Dynamics in an Isolated Perfused Rat Heart," *Biol. Cybern.* **61**, 371–378 (1989).
11. R. E. Kleiger, J. T. Bigger Jr., M. S. Bosner, M. K. Chung, J. R. Cook, L. M. Rolnitzky, R. Steinman and J. L. Fleis, "Stability Over Time of Variables Measuring Heart Rate Variability in Normal Subjects," *Am. J. Cardiol.* **68**, 626–630 (1991).
12. J. T. Bigger, J. L. Fleiss, R. C. Steinman, L. M. Rolnitzky, R. E. Kleiger and J. N. Rottman, "Frequency Domain Measures of Heart Period Variability and Mortality after Myocardial Infarction," *Circulation* **85**, 164–171 (1992).
13. M. Meesmann, J. Boese, D. Chialvo, P. Kowallik, W. R. Bauer, W. Peters, F. Grüneis and K.-D. Kniffki, "Demonstration of $1/f$ Fluctuations and White Noise in the Human Heart Rate by the Variance-Time-Curve: Implications for Self-Similarity," *Fractals* **1**, 000–000 (1993b) in press.
14. K. Kawahara, Y. Yamauchi, Y. Nakazono and Y. Miyamoto, "Spectral Analysis on Low Frequency Fluctuation in Respiratory Rhythm in the Decerebrate Cat," *Biol. Cybern.* **61**, 265–270 (1989).
15. M. J. Campbell and B. W. Jones, "Cyclic Changes in Insulin Needs of an Unstable Diabetic," *Science* **177**, 889–891 (1972).
16. M. Yamamoto, H. Nakahama, K. Shima, K. Aya, T. Kodama, H. Mushiake and M. Inase, "Neuronal Activities During Paradoxical Sleep," *Adv. Neurolog. Sci.* **30**, 1010–1022 (1986).
17. F. Grüneis, M. Nakao and M. Yamamoto, "Counting Statistics of $1/f$ Fluctuations in Neuronal Spike Trains," *Biol. Cybern.* **62**, 407–413 (1990).
18. K.-D. Kniffki, M. K. C. Mengel and C. Vahle-Hinz, "A New Tool to Characterize Ongoing Neuronal Discharge: Counting Statistics of f^{-b} -Fluctuations," *Soc. Neurosci. Abstr.* **18**, 1393 (1992).
19. M. K. C. Mengel, C. Vahle-Hinz and K.-D. Kniffki, "Is the Activity of Neurones in the Somatosensory Thalamus an Expression of Self-organized Criticality?" *Europ. J. Neurosci. Suppl.* **5**, 138 (1992).
20. M. C. Teich, "Fractal Character of the Auditory Neural Spike Train," *IEEE Trans. Biomed. Eng.* **36**, 150–160 (1989).
21. K. Christensen, Z. Olami and K. Bak, "Deterministic $1/f$ Noise in Nonconservative Models of Self-organized Criticality," *Phys. Rev. Lett.* **68**, 2417–2420 (1992).
22. P. Bak, "Simulation of Self-organized Criticality," *Physica Scripta.* **T33**, 9–10 (1990).
23. F. Moss, "Stochastic Resonance," *Ber. Bunsenges. Phys. Chem.* **95**, 303–311 (1991).
24. A. Longtin, A. Bulsara and F. Moss, "Time-interval Sequences in Bistable Systems and the Noise-induced Transmission of Information by Sensory Neurons," *Phys. Rev. Lett.* **67**, 656–659 (1991).
25. S. B. Lowen and M. C. Teich, "Fractal Renewal Processes Generate $1/f$ Noise," *Phys. Rev.* **E47**, 992–1001 (1993).
26. B. B. Mandelbrot, *The Fractal Geometry of Nature* Freeman, New York (1983).