A Layered Description of ATM Cell Traffic Streams and Correlation Analysis

O. GIHR* and P. TRAN-GIA**

* Centre for Teletraffic Research, Bond University, Gold Coast, Qld 4229, Australia

** University of Würzburg, Am Hubland, W-8700 Würzburg, Germany

Abstract *This paper presents an approach to describe traffic processes in ATM environments. Using a discrete-time Markov chain to describe the cell process dynamics, we derive an algorithm to calculate the correlation function of the traffic process. As an example the correlation properties of the well-known two-state process are investigated. We use a hierarchical characterization of discrete-time traffic processes to capture the short-term and the long-term dependencies of process segments as well as different time scales according to cell, burst, dialog and call layers of traffic streams in ATM systems. It is shown by comparison of the process description techniques using the Index of Dispersion of Count (IDC) and using the Correlation Function (CF), that the CF gives significant additional insight to understand the short- and long-term dependencies of traffic processes in ATM environments.*

1 Introduction

The characterization of traffic streams is an essential factor in performance analysis of systems in ATM environments. It forms the first step towards accurate and appropriate models of such systems. In contrast to the conventional telephony, where most of the models are sufficiently accurate considered in continuous-time domain offering Poisson or renewal processes, the ATM traffic processes are different and are of a more complex nature.

It is obvious and convenient to model ATM traffic streams by means of discrete-time processes due to the slotted cell transmission time. As illustrated in Fig. 1, the model can be observed on call (or connection) layer, on burst layer or on cell layer (see also [1]). According to this, the time resolution differs in order of magnitude, i.e. in seconds, milliseconds or microseconds. Thus an accurate ATM traffic model must be characterized in discrete-time domain. Further, it must be flexible enough to govern the time scales mentioned. Clearly, it is a non-trivial task to develop a single description technique to characterize all properties of traffic processes on different time resolution levels.

The work described herein was done while Prof. Phuoc Tran-Gia was visiting Bond University.

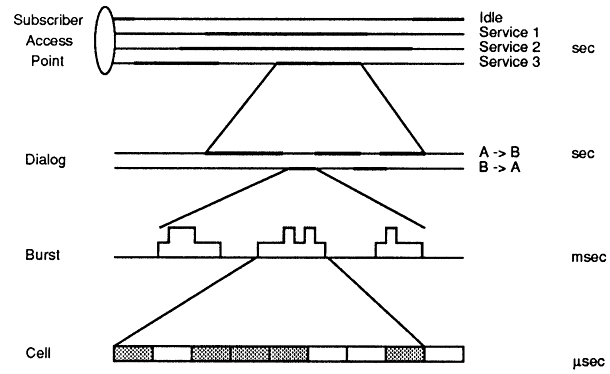

Figure 1: *Layered ATM Traffic Stream Description*

In conjunction with the time resolution problem, the dependency and the correlation between process segments turn out to be important factors influencing the accuracy of ATM traffic models (see [11, 5, 9, 4]). Besides short-term correlation effects, long-term dependencies are topics of interest in variable-bit-rate video traffic. Two classes of models have been introduced to describe these effects:

- Autoregressive processes and
- Markov chains in discrete- or continuous-time.

The autoregressive processes are mathematically simple to describe, but their analytical treatment is rather difficult. Therefore, the Markov chain models are preferable from the analysis viewpoint. Maglaris et. al. [5] used an autoregressive process as simulation model and a continuous-time discrete-state Markov chain as analytical model to describe medium-term dependencies of the bit rates of variable-bit-rate video sources at the video-frame level.

Nomura et. al. [6] used the autoregressive process to describe the medium-term dependencies at the video-frame level and an overlaid continuous-time Markov chain to model scene changes with long-term dependencies. In [7] and [10] discrete-time Markov chains have been used to model variable-bit-rate video. In Zukerman and Potter [13] an autoregessive model on the frame level and a discrete-time two state Markov chain on the cell level within a frame have been employed. An approach using a discrete-time Markov chain to model the cell traffic stream can be found in Ramaswami and Latouche [8].

In general, discrete-time approaches can be separated in the following categories:

(i) Description of all-layer traffic processes using one discrete-time Markov chain

(ii) Considerations of the *Index of Dispersion of Counts (IDC)* or *Index of Dispersion of Intervals (IDI)* to describe short-term and long-term process dependencies

(iii) Considerations of correlation functions to describe inter-segment dependencies. They enable the graphical recognition of the correlation structure of the traffic process. This is one of the major objectives in this paper.

In this paper, an approach to describe traffic processes in ATM environments using correlation functions will be discussed. Starting with a discrete-time Markov chain characterization of the cell process dynamics, we arrive at an algorithm to calculate the *Correlation Function* CF of the traffic process. The well-known two-state process (or on-off process) is taken as an example. This correlation function description technique is then compared to the description technique using the *Index of Dispersion of Counts* IDC. Furthermore, to govern the problem of time resolution differences, we use a hierarchical characterization of discrete-time traffic processes and derive the correlation function of a two layer process.

2 Layered Characterization of ATM Traffic

2.1 Multilayer Description with Discrete-Time Markov Chain

2.1.1 The Basic Single Layer Description

First we look at an ATM traffic stream on the cell layer (cf. Fig. 2). For the characterization the general Markov chain description proposed in [8] is used. The basic Markov chain consists of a number n of states, which are subdivided into ν active and ω silent states. The states can be grouped into sets, e.e. A_1, A_2 and S. The state transition behaviour is completely described by the state transition matrix $\underline{\underline{Q}}$ with elements $q_{i,j}$. The time is slotted by the cell duration. After each slot a state transition will take place. During each transition the following will occur:

(i) if the Markov chain was in an active state, a cell will be generated,

(ii) if the Markov chain was in a silent state, the state transition will take place without any cell generation.

Ramaswami and Latouche [8] showed that this process description technique is quite versatile and is able to characterize complex processes with arbitrary short-term and long-term dependencies and correlations. In some cases however, the number of states required can be very large.

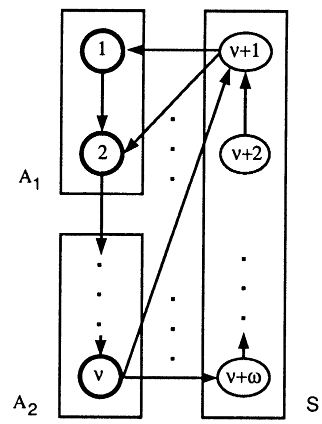

Figure 2: *Description Technique on the Cell Layer*

Thus, a disadvantage of this technique is, that in case of long-term correlations the transition matrix tends to become large in size. It is obvious that to describe correlations for a variable-bit-rate video scene, e.g., the single-layer Markov chain description of cell traffic is not very convenient.

2.1.2 Multilayer Description

To illustrate the multilayer description and cell clustering effect we take the example in Fig. 1, where four traffic layers are depicted: call, dialog, burst and cell layer. Without any knowledge of the traffic layers the arrival process is observed as a simple alternation process between the two phases: cell generation phase and silent phase.

However, the length of these phases depend on higher layer processes as shown in the example of Fig. 3. There exist different types of silent phases, which can be used to describe short-, medium- and long-term dependencies and correlations.

On cell layer with a resolution in microseconds, the cell generation phase, which is simply a cell cluster of length A_0, consists of a number of contiguous cells. The corresponding silent phase is S_0. A cluster of A_0 and S_0 constructs a cell burst with length A_1.

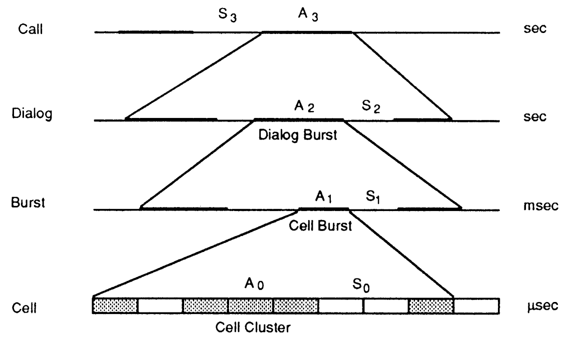

Figure 3: *Layered Traffic Stream Description of a Call*

By observing the process on a longer time scale, longer silent phases become recognizable, which separate the cell bursts. These silent phases of a higher description layer are referred to as interburst silence S_1. A process characterization using S_1 and A_1 will then be able to describe properties of ATM traffic streams, which can be seen first on microsecond resolution level. Continuing the layered description to dialog and call layers we arrive at connection duration A_3 and intercall interval S_3, which can be used to describe the subscriber behaviour on a time resolution level of seconds.

2.2 Example

The process described above can be represented in a layered model as depicted in Fig. 4. From this complete description of a process the description of the layer of interest can be derived. If the interest is on the cell layer this layered description is transformed into one Markov chain with the time scale of the cell layer. If we assume that the silent states of all layers have only a state transition to itself and the active state, then the dissolution of the burst layer can be done as depicted in Fig. 5. Following this procedure up to the call layer we get the Markov chain description as illustrated in Fig. 6.

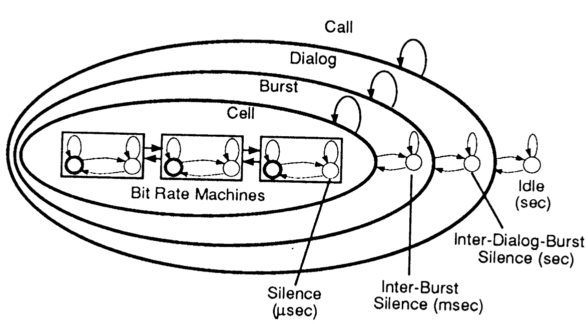

Figure 4: *Markov Chain Model of a Call*

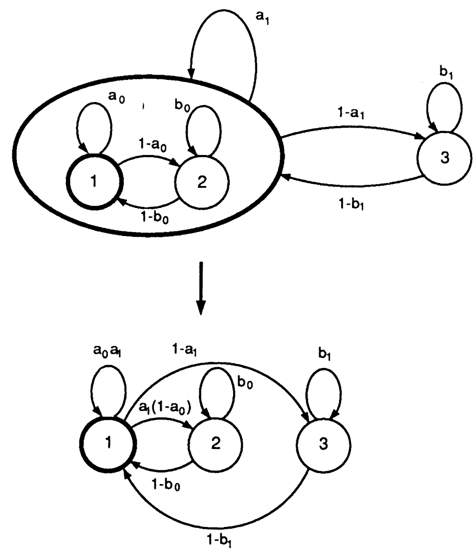

Figure 5: *Dissolution of the Burst Layer*

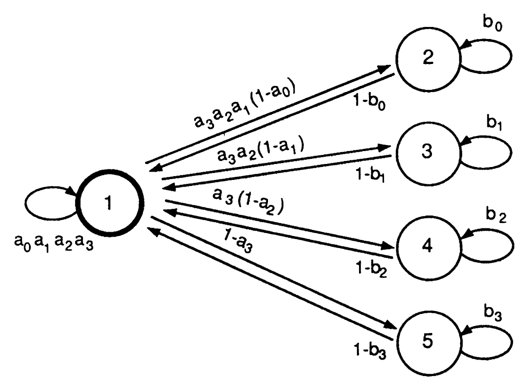

Figure 6: *Single Markov Chain of the Process at the Cell Layer*

3 Correlation Analysis

3.1 Correlation Function

In the next subsections we will introduce the correlation function (auto-correlation function) of a random stationary sequence $x(k)$. First we restrict the values of $x(k)$ to the integer set $[0, 1]$ representing a present cell and a missing cell, i.e. the states active and silent, respectively. This will be generalized to a range of $x(k)$ of $[0, 1]$ representing the utilization of the line during a specified time period.

3.1.1 Cell Layer

The cell traffic stream is represented as a random stationary sequence $x(k)$ with the values 0 and 1 representing a cell and a missing cell, respectively. The correlation function $R(m)$ is accordingly defined as :

$$R(m) = P\{x(k) = 1, x(k+m) = 1\} \tag{1}$$

This can be interpreted as the probability that an outside observer sees a pair of active cells, where the cells are separated by $m-1$ cells. The cell traffic stream is modelled with a discrete-time Markov chain. The states are marked either as *active* or *silent*. As mentioned, if the Markov chain is in an *active* state then a cell is generated at the state transition. If the Markov chain is in a silent state then no cell is generated at the state transition.

Let be $\underline{\pi}$ the outside observer state probabilities of the Markov chain, which are separated into two parts: active states $(1..\nu)$ and silent states $(\nu+1..\nu+\omega)$. With $\underline{\underline{Q}} = \{q_{i,j}\}$ as the state transition probability matrix and $\underline{\underline{Q}}^m = \{q_{i,j}^{(m)}\}$ the m-step state transition probability matrix, the correlation function can be expressed as:

$$R(m) = \sum_{i=1}^{\nu} \pi_i \sum_{j=1}^{\nu} q_{i,j}^{(m)} \tag{2}$$

3.1.2 Burst, Dialog and Call Layer

The time is slotted due to the burst, dialog or call layer with an appropriate slot duration. Analogously, the random stationary sequence $x(k)$ represents the utilization of the line with a range $[0,1]$. The correlation function is defined as:

$$R(m) = \int_{z=-\infty}^{\infty} \int_{y=-\infty}^{\infty} yz p_{k,k+m}(y,z) dydz \tag{3}$$

with

$$\begin{aligned} p_{k,k+m}(y,z) &= p_m(y,z) \\ &= P\{x(k) = y, x(k+m) = z\} \end{aligned} \tag{4}$$

In the Markov chain model an activity value a_i is assigned to state i (i=1..n). The values a_i can be obtained from the description in conjunction with the analysis of the next lower traffic description layer. The correlation function becomes then :

$$R(m) = \sum_{i=1}^{n} a_i \pi_i \sum_{j=1}^{n} a_j q_{i,j}^{(m)} \tag{5}$$

3.2 Index of Dispersion

The index of dispersion I of a random variable T is known as the ratio $I = VAR[T]/E[T]$. It is often used as a measure of burstiness of the traffic process. In the following we will first focus the attention to the *Number of Events* in an observed time interval of a traffic stream and then we discuss the *Index of Dispersion of Counts* of the corresponding Markov chain.

3.2.1 Number of Events

Observing the traffic stream in discrete-time, each slot either contains a cell or is empty, i.e. a cell is present or missing. The present cells are regarded as events of the process and are counted. The content of the counter after observing m consecutive slots is assigned to a random variable $N(m)$ having the range of $[0..m]$. The corresponding distribution is $P\{N(m) = k\} = x_k^{(m)}$.

For the process of counts starting in state i of a Markov chain the probabilities $P\{N_i(m) = k\} = x_{i,k}^{(m)}$ can be expressed recursively by :

$$x_{i,k}^{(m)} = \begin{cases} x_{i,0}^{(m-1)} q_{i,S}^{(m)} & ; k = 0 \\ x_{i,k}^{(m-1)} q_{i,S}^{(m)} + x_{i,k-1}^{(m-1)} q_{i,A}^{(m)} & ; 0 < k < m \\ x_{i,m-1}^{(m-1)} q_{i,A}^{(m)} & ; k = m \end{cases} \tag{6}$$

with

$$q_{i,A}^{(m)} = \sum_{j=1}^{\nu} q_{i,j}^{(m)} \quad \text{and} \quad q_{i,S}^{(m)} = \sum_{j=\nu+1}^{\nu+\omega} q_{i,j}^{(m)}. \tag{7}$$

$q_{i,j}^{(m)}$ are again the m-step transition probabilities with

$$q_{i,j}^{(0)} = \begin{cases} 1 & ; i = j \\ 0 & ; otherwise \end{cases} . \tag{8}$$

As mentioned above, the states $1..\nu$ indicate the active states and $\nu+1..\nu+\omega$ the silent states. The mean and variance are given by :

$$E[N_i(m)] = \sum_{k=1}^{m} k x_{i,k}^{(m)} = \sum_{j=1}^{m} q_{i,A}^{(j)} \tag{9}$$

$$\begin{aligned} VAR[N_i(m)] &= \sum_{k=0}^{m} (k - E[N_i(m)])^2 x_{i,k}^{(m)} \\ &= \sum_{j=1}^{m} q_{i,A}^{(j)} q_{i,S}^{(j)} \end{aligned} \tag{10}$$

3.2.2 Index of Dispersion of Counts (IDC)

The variance of the *Number of Events N(m)* of a (periodic or aperiodic) Markov chain is defined by the sum of the variances of $N_i(m)$ of each state weighted by the outside observer state probability π_i.

$$\begin{aligned} IDC(m) &= \frac{VAR[N(m)]}{E[N(m)]} \\ &= \frac{\sum_{i=1}^{\nu+\omega} \pi_i VAR[N_i(m)]}{\sum_{i=1}^{\nu+\omega} \pi_i E[N_i(m)]} \\ &= \frac{\sum_{i=1}^{\nu+\omega} \pi_i VAR[N_i(m)]}{ma} \end{aligned} \tag{11}$$

with $a = \sum_{i=1}^{\nu} \pi_i$. For a periodic Markov chain the index of dispersion of counts will be zero, if the Markov chain has one single route, e.g. the set of possible states at each step is one. This property holds regardless of the complexity of the Markov chain and the distribution of the used and unused cells.

3.3 Examples

3.3.1 Two-State Process

In this subsection we take as example the well-known two-state process as depicted in Fig. 7. The transition matrix $\underline{\underline{Q}}$ is given by :

$$\underline{\underline{Q}} = \begin{pmatrix} q_{AA} & q_{AS} \\ q_{SA} & q_{SS} \end{pmatrix} = \begin{pmatrix} p & 1-p \\ t & 1-t \end{pmatrix} \tag{12}$$

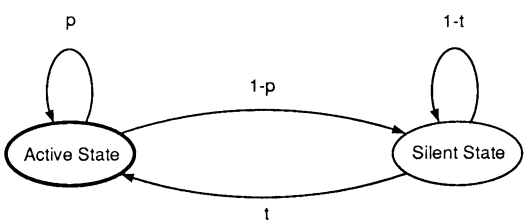

Figure 7: *Model of the Two-State Process*

Accordingly, the outside observer state probabilities are :

$$\underline{\pi} = (\pi_A, \pi_S)^T = \begin{cases} \left(\frac{t}{1-p+t}, \frac{1-p}{1-p+t}\right)^T & ; 1-p+t \neq 0 \\ (0.5, 0.5)^T & ; 1-p+t = 0 \end{cases} \tag{13}$$

The m-step transition matrix can be obtained analytically as (see Hunter [3]) :

$$\begin{aligned} \underline{\underline{Q}}^m &= \begin{pmatrix} q_{AA}^{(m)} & q_{AS}^{(m)} \\ q_{SA}^{(m)} & q_{SS}^{(m)} \end{pmatrix} \\ &= \begin{pmatrix} \pi_A + \pi_S(p-t)^m & \pi_S - \pi_S(p-t)^m \\ \pi_A - \pi_A(p-t)^m & \pi_S + \pi_A(p-t)^m \end{pmatrix} \end{aligned} \tag{14}$$

This leads to the correlation function :

$$\begin{aligned} R(m) &= \frac{t}{(1-p+t)^2}(t + (1-p)(p-t)^m) \\ &= \pi_A^2 + \pi_A(1-\pi_A)(p-t)^m \end{aligned} \tag{15}$$

The limits are $R(0) = \pi_A$ and for an aperiodic Markov chain $R(\infty) = \pi_A^2$. For a periodic Markov chain the *Correlation Function* CF is as well periodic. We consider in Fig. 8 the four cases

i) $p = t = 0.5$, i.e. the two-state process represents a Bernoulli input process with probability 0.5

ii) $p = 0, t = 1$, i.e. a cell appears every second slot

iii) $p = 0.9, t = 0.1$, i.e. the probability to stay in the same state is 9 times higher than to escape the current state and

iv) $p = 0.1, t = 0.9$, i.e. the probability to escape the current state in 9 times higher than to stay in the state

(see also Table 1). Note that for all four cases $\pi_A = \pi_S = 0.5$.

For the case i) and ii) the *Correlation Function* can easily be estimated. Case iii) indicates no oscillation, since the tendency to stay in the current state is higher than a state change. Case iv) leads to an oscillation of the *Correlation Function*, since the probability to escape the current state is high and the tendency to return to the same state in the second step is dominant. As depicted, this effect of oscillation, i.e. the periodic dependency, can be observed very clearly using the *Correlation Function* rather than using the *Index of Dispersion of Counts*, as we show later in this section.

The *Index of Dispersion of Counts* can be expressed again in a closed form :

$$\begin{aligned} IDC(m) &= \frac{VAR[N(m)]}{E[N(m)]} \\ &= \left(\pi_A \sum_{j=1}^{m} q_{AA}^{(j)} q_{AS}^{(j)} + \pi_S \sum_{i=1}^{m} q_{SA}^{(i)} q_{SS}^{(i)}\right) / m\pi_a \\ &= \pi_S \left(1 - \frac{(p-t)^2}{m} \frac{1-(p-t)^{2m}}{1-(p-t)^2}\right) \end{aligned} \tag{16}$$

	i Bernoulli	ii Periodic	iii	iv
p	0.5	0	0.9	0.1
t	0.5	1	0.1	0.9

Table 1: *Parameters of the Two-State Process*

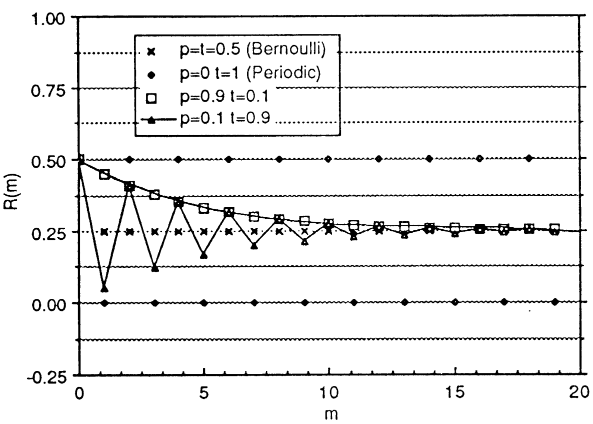

Figure 8: *Correlation Function of the Two-State Process*

For large m the index of dispersion of counts converges to $IDC(\infty) = \pi_S$.

It should be noted here that the index of dispersion of counts and the correlation function contain different information and can not express each other in the general case. However, for the two-state process it is possible to express the index of dispersion of counts by the correlation function defined above, which are based on the observation of present cells. Additionally, a correlation function for the missing cells can be derived.

In Fig. 9 the IDC is depicted for the parameters listed in Table 1. Note that the cases iii) and iv) result in the same IDC-function.

3.3.2 Two-Layer Process

We investigate in this subsection the two-layer process depicted in Fig. 10 with two different time scales. On the higher layer (macro layer) a two-state model is defined. The two states $S_{2,1}$ and $S_{2,2}$ represent an active macro-state and a silent macro-state. Within the state $S_{2,1}$ a two-state cell layer model is defined with an active cell state and a silent cell state. Within the macro state $S_{2,2}$ a single silent cell state is defined.

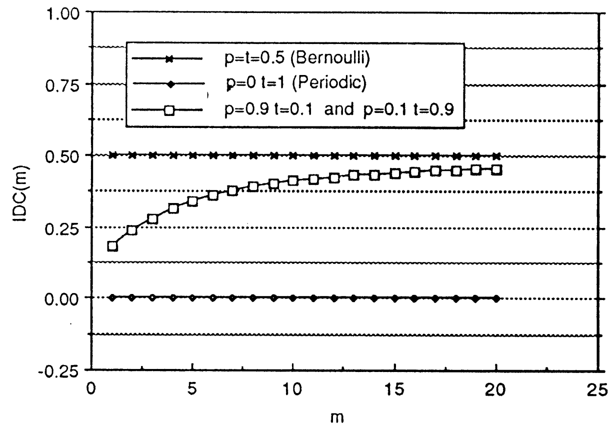

Figure 9: *Index of Dispersion of Counts of the Two-State Process*

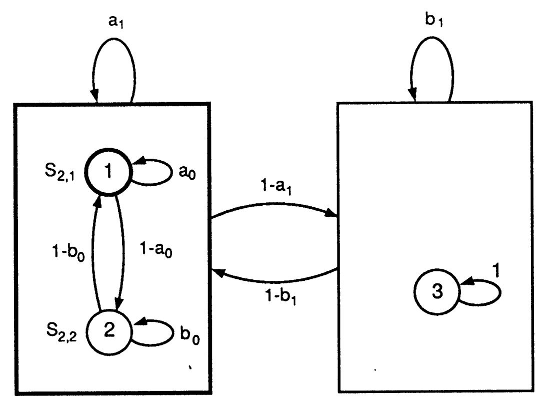

Figure 10: *Two-Layer Model*

The times scales are chosen to fit a factor of ϵ=100 with the corresponding slot durations for the macro-state and the cell-state model time scales. The transition probabilities are depicted in Table 2. The parameters are chosen to represent a burst process at the macro layer and a nearly periodic process on the cell layer in the macro state $S_{2,1}$.

This layered description can be transformed into a single aggregated Markov chain as depicted in Fig. 11. The transition probabilities of the macro layer are modified to the cell layer time scale (e.g. $a_1^* = 1 - (1 - a_1)/(1 + a_1(\epsilon - 1)))$ to get the same mean time in the previous macro-states. The macro states can then be eliminated by assuming that i) the macro-state is left from each cell layer state with the same probability and ii) by entering a macro-state, each state of the cell layer is entered with the outside observer state probability. The modified transition probabilities are depicted in Table 3.

$a_0 = 0.1$	$b_0 = 0.1$
$a_1 = 0.9$	$b_1 = 0.9$

Table 2: *Transition Probabilities of the Two Layer Model*

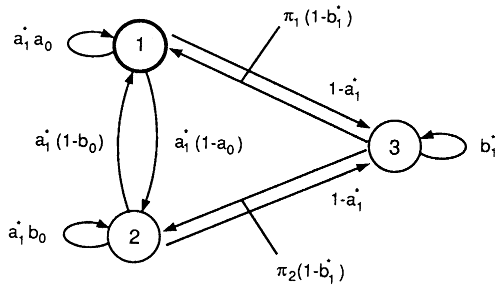

Figure 11: *Representation of the Two-Layer Model in a Single Markov Chain*

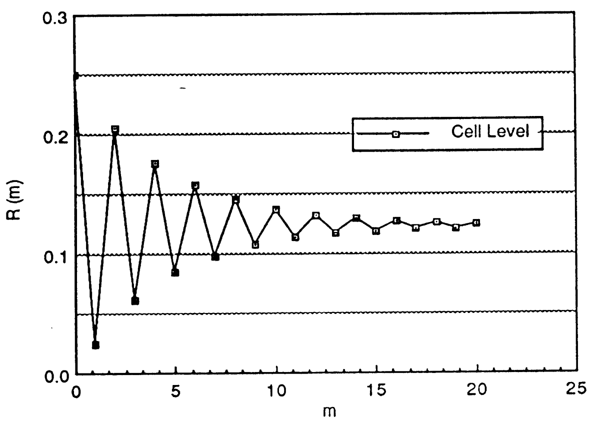

Figure 12: *Short-Term Correlation Function of the Two-Layer Model in a Single Markov Chain*

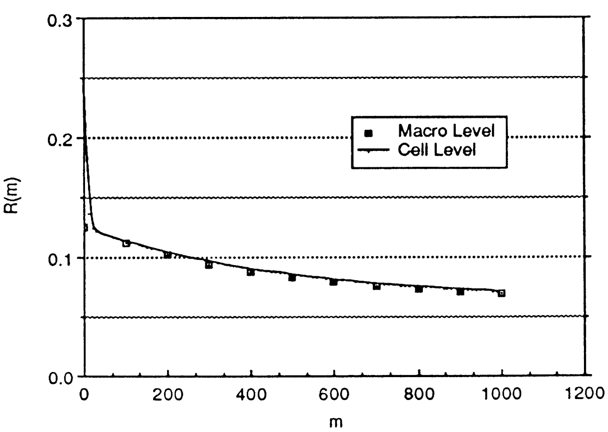

Figure 13: *Long-Term Correlation Function of the Two-Layer Model in a Single Markov Chain*

$a_0 = 0.1$	$b_0 = 0.1$
$a_1^* = 0.999$	$b_1^* = 0.999$

Table 3: *Modified Transition Probabilities of the Two Layer Model*

In Figs. 12 and 13 the correlation functions of the Markov chain is depicted for the short-term and the long-term correlations, respectively. Additionally, in Fig. 13 the *Correlation Function* obtained from the macro layer description according section 3.1.2 is depicted. The same interpretation as in Fig. 8 case iv) can be done.

4 Conclusion and Future Research

In this paper a hierarchical modelling approach of discrete-time traffic processes has been described, where a basic Markov chain description technique is employed in conjunction with the correlation analysis. The approach can be used in investigations of traffic processes, e.g. in ATM environments, where time scales at cell, burst, dialog and call traffic description layers differ in order of magnitude. Due to the slotted-time nature of the considered processes discrete-time Markov chains are used. The use of the correlation function has been compared with the process description by means of the index of dispersion of counts, where aspects concerning the visualization and the characterization of dependencies and correlation structures of process segments have been discussed. The two-state process was taken as an example.

The hierarchical modelling approach has been also demonstrated observing a two-layer process with two time scales. The Markov chain from the hierarchical model is derived and the correlation function illustrating short-term and long-term correlation is depicted.

It can be seen by the examples that the *Correlation Function* is able to visualize the dependencies and the correlation structure of process segments in a better way than the *Index of Dispersion of Counts*. Additionally, the correlation function has the advantage that it can be measured directly out of the process using e.g. well-developed standard signal processing equipments.

The modelling technique has now to be applied to more complex and more realistic traffic streams. A procedure has to be developed, which is able to derive the parameters of the Markov chain model from the measured correlation function, e.g. of a video codec source. To complete the correlation analysis the abilities of the *spectral density function* can be inspected. Subsequent to the correlation analysis of a traffic stream the multiplexing of several traffic streams with additional change of the time scale could be considered.

Acknowledgements

We wish to express our thanks to Prof. Les Berry, James Gordon and Frank Hübner for helpful and stimulating discussions. We also gratefully acknowledge the partial support of Telecom Australia in this project.

References

[1] J. Filipiak, " Structure of traffic flows in multiservice networks" , Autralian Telecommunication Research (ATR), Volume 23, Number 1, 1989, pp. 5-18.

[2] J.J. Gordon, " A traffic performance model for ATM multiplexers" , Fourth Australian Teletraffic Research Seminar, Bond University, December 1989.

[3] J. J. Hunter, " Mathematical Techniques of Applied Probability, Volume 2 – Discrete Time Models: Techniques and Applications" , Academic Press, New York, 1983.

[4] R. Kishimoto, Y. Ogata, F. Inumaru, "Generation interval distribution characteristics of packetized variable rate video coding data streams in an ATM network" , IEEE JSAC, Vol. 7 (1989) 5, pp. 833-841.

[5] B. Maglaris, D. Anatassiou, P. Sen, G. Karlsson, J. Robbins, "Performance analysis of statistical multiplexing for fast packet video sources" , IEEE GLOBECOM'87, 47.8, Nov. 1987, pp. 1890-1899.

[6] M. Nomura, T. Fujii, N. Ohta, "Basic characteristics of variable rate video coding in ATM environment" , IEEE JSAC, Vol. 7 (1989) 5, pp. 752-760.

[7] P. Pancha, M. E. Zarki, "Modelling video sources for resource allocation in ATM based B-ISDN" , Third Intern. Workshop on Packet Video; Morristown, NJ, USA, March 1990.

[8] V. Ramaswami, G. Latouche, "Modeling packet arrivals from asynchronous input lines" , Proceedings of ITC 12, Torino, June 1988, paper 4.1A.5.

[9] P. Sen, B. Maglaris, N.-E. Rikli, D. Anastassiou, "Models for packet switching of variable bit-rate video sources" , IEEE JSAC, Vol. 7 (1989) 5 , pp. 865-869.

[10] J.J. Shin, J. Kim, "An analysis of the packet video multiplexing by a 3-state discrete-time markov chain model" , Third Intern. Workshop on Packet Video, Morristown, NJ, USA, March 1990 .

[11] K. Sriram, W. Whitt, "Characterizing superposition arrival processes in packet multiplexers for voice and data" , IEEE JSAC, Vol. 4 (1986) 6, pp. 833-846 .

[12] W. Verbiest, L. Pinnoo, "A variable bit rate video codec for asynchronous transfer mode networks" , IEEE JSAC, Vol. 7 (1989) 5, pp. 761-770.

[13] M. Zukerman, P. G. Potter, "The effect of eliminating the standby state on DQDB performance under overload" , Intern. Journal of Digital and Analog Cabled Systems, Vol. 2 (1989), pp. 179-186.