

Simulation of Instationary Processes for Performance Evaluations of Switching Systems

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Abstract

A simulation method for investigations of transient behaviour of queueing systems is described in this paper. The method is based on a modification of the event-by-event simulation technique where arrivals according to time-dependent, generalized Poisson processes are considered. Applications of the method to performance evaluation of switching systems are discussed where the transient behaviour of the $M^{[X]}(t)/M/1$ queueing model is investigated. The more general queueing system $M(t)/G/n$ and its time-dependent responses to instationary traffic patterns are also considered in the last part of the paper.

1. STATEMENT OF THE PROBLEM

In performance investigations of telecommunication systems, e.g. stored program controlled telephone switching systems, the event-by-event simulation technique is a well-established method [1]. This technique is applied to validate analytically obtained results or to enable the model investigation when an analytic method is not available.

The most simulation studies in the literature deal with model investigations under stationary traffic conditions. These studies allow considerations of dimensioning aspects as well as performance evaluations of telecommunication systems. However, in order to investigate the system performance under overload conditions, modelling approaches for instationary system loads must be taken into account. The instationary traffic offered can be, e.g. represented by means of a generalized Poisson process with a time-dependent arrival rate $\lambda(t)$. Using this approach for traffic characterization, investigations of telephone switching systems are given in [4, 5] which provide instationary system characteristics, such as system response to pulse-form overload patterns, overload handling efficiency, etc... For this class of studies, analysis and simulation methods for instationary conditions are required.

This paper presents an exact method for the simulation of instationary random processes with time-dependent Poissonian input traffic. The method will be described in chapter 2 and applications to switching system investigations and to general queueing systems are given in chapter 3.

2. SIMULATION METHOD OF INSTATIONARY PROCESSES

In the following a simulation technique for instationary system investigations, especially for systems with generalized Poisson process input, will be described [5]. The method is a modification of the well-known event-by-event simulation where two aspects are considered for the nonstationary case: the event generation technique according to the generalized Poisson process and the measurement technique for transient system characteristics.

2.1 Event Generation for the Generalized Poisson Process

The event generation method described here is based on the standard method of generation by inversion corresponding to a numerical probability transformation technique [1]. Modifications of this technique for the generalized Poisson process can be found in [2] and [3]. The basic procedure of this technique is to first generate a uniformly-(0,1) distributed random variate z . The random variate T_A according to the interarrival distribution function $F_A(t)$ can then be determined as illustrated in Fig.2.1.

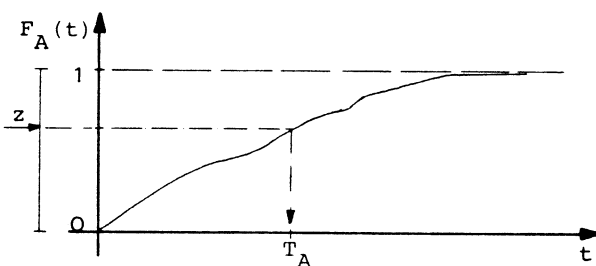


Fig.2.1 Inversion technique for event generation.

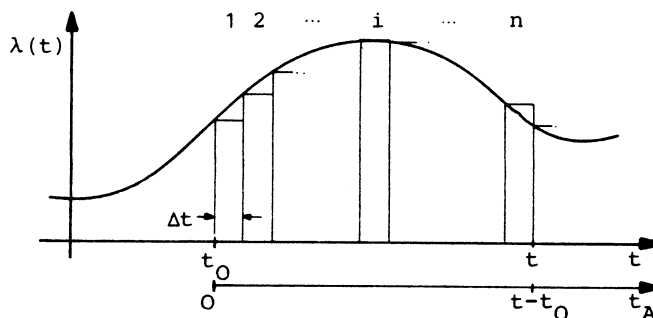


Fig.2.2 Derivation of the time-dependent interarrival distribution function by time discretization.

In the case of $\lambda(t)$, a time-dependent interarrival distribution function $F_{t_0}^{t_A}$ must be taken into account, conditioning on an arrival at time t_0 . The derivation will be outlined in the following.

As shown in Fig. 2.2, the time between the last arrival t_0 and the observation instant t is discretized into n intervals of length Δt , where the Poissonian arrival rate during the i -th interval is approximated to be constant :

$$\lambda_i = \lambda(t_0 + (i-1)\Delta t) , i=1,2,\dots,n. \tag{2.1}$$

Since the probability that no arrivals occur in the i -th interval is $\exp(-\lambda_i \Delta t)$, the probability for having no arrivals in the interval (t_0, t) can be written as

$$\Pr\{T_{A,t_0} > t_A\} = \prod_{i=1}^n \exp(-\lambda_i \Delta t) = \exp(-\Delta t \cdot \sum_{i=1}^n \lambda_i) \tag{2.2}$$

where T_{A,t_0} denotes the time-dependent random variable for the interarrival time at the time t_0 instant t_0 .

Finally, for $n \rightarrow \infty$ and $\Delta t \rightarrow 0$ in eqn.(2.2), the complementary time-dependent interarrival distribution function can be derived :

$$F_{t_0}^C(t_A) = \Pr\{T_{A,t_0} > t_A\} = \lim_{\substack{n \rightarrow \infty \\ \Delta t \rightarrow 0}} \exp(-\Delta t \cdot \sum_{i=1}^n \lambda_i) = \exp\left(-\int_{t_0}^{t_0+t_A} \lambda(t) \cdot dt\right)$$

and the time-dependent interarrival distribution function is given by

$$F_{t_0}^{t_A} = 1 - \exp\left(-\int_{t_0}^{t_0+t_A} \lambda(t) \cdot dt\right). \tag{2.3}$$

Using the expression given in eqn.(2.3), the inversion technique for the stationary case (Fig. 2.1) can be used to obtain the next event for the instationary Poisson process with arbitrary time-dependent rate $\lambda(t)$.

In modelling approaches for overload control investigations in telecommunication systems, especially in telephone switching systems, instationary traffic in the form of short-term overload is often considered. This can be modelled by means of the following time-dependent Poissonian rate :

$$\lambda(t) = \lambda_1 + \lambda_{OL}(t) \quad (2.4)$$

normal traffic level
 overload traffic
 instationary traffic

As given in eqn.(2.3), conditioning on the last arrival at time t_0 , the next arrival will be at the time epoch $t_0 + t_A$ according to the following time-dependent interarrival distribution function

$$F_{t_0}(t_A) = 1 - \exp(-\lambda_1 \cdot t_A - K(t_0, t_A))$$

where $K(t_0, t_A)$ represents a time-dependent correcting function

$$K(t_0, t_A) = \int_{t_0}^{t_0+t_A} \lambda_{OL}(t) \cdot dt \quad (2.5)$$

2.2 Organization of Transient Simulations

The simulation of a transient process usually begins at a starting point with a starting condition, e.g. the system is empty or in a predefined stationary or quasi-stationary state. Based on the starting condition which is usually given in the form of an initial probability vector, the process is simulated under instationary conditions until an instant, say t_1 , where the process is in another stationary state or when the time interval of interest is exceeded. As illustrated in Fig.2.3, a single transient simulation task as described will be called an *elementary test*. Each elementary test will start at the same system initial condition. A *simulation run* consists of a number of *part-tests*; each part-test is composed of a large number of elementary tests. The simulation program must be implemented in such a way to ensure the independency between elementary tests.

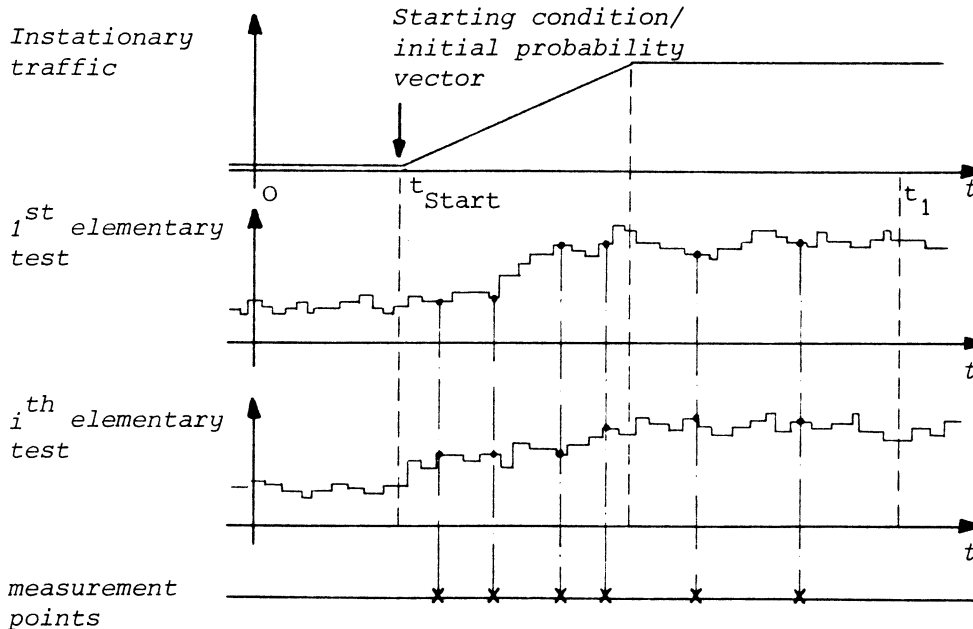


Fig. 2.3 Elementary tests and measurement technique of instationary simulation

The measurement of values of interest or system characteristics can be done by means of standard technique [1] at measurement points (Fig.2.3) which can arbitrarily be chosen depending on the particular investigation requirements. The statistical analysis of a simulation run is done using the ensemble averages of elementary tests. For measurements of waiting time or flow time in the process, virtual test-customers must be inserted in the current process and their fates, i.e. their waiting time or flow time, are observed.

In the applications described in the next chapter the Student-t-test technique is employed where for each measurement point the system characteristic to be investigated is given in the form of a sample mean and a related confidence interval which are calculated out of part-test results.

3. EXAMPLES FOR APPLICATION

3.1 Overload Performance of Switching Systems

a) Model Description

Analytical estimation of overload control performance of stored program control (SPC) switching systems is a subject which has been addressed in some recent studies [4, 5] whereby the instationary simulation technique is used for the purpose of validation. The modelling approach which will be resumed below is investigated in more detail in [4].

The model deals with the interdependency between telephone customers and a switching system where the dynamical system performance, i.e. the system throughput measured in terms of the call completion rate, is taken into account. The queueing model used is of type

$$M^{[x]}(t)/G/1$$

where

- $M(t)$ models the instationary arrival process (generalized Poisson process) with rate $\lambda(t)$ for telephone calls under overload conditions
- the batch process stands for the number of tasks or telephonic events (subcalls) generated for call handling in the processing unit. The batch size is dependent on the actual system state upon the call arrival instant; depending on the batch size (number of subcalls), the call completion characteristic is estimated
- the single server models the processing unit which handles subcalls. In the analytical studies, a Markovian server is considered, while for the instationary simulation, arbitrarily distributed service times can be taken into account.

b) Instationary Behaviour under Overloads

Some results will be given in the following to show the accuracy of the instationary simulation. The time axis is normalized by the mean service time h , and the normalized overload traffic $\rho_0(t)$ is considered as short-term, time-continuous triangular pulse patterns. Fig.3.1 shows a comparison between simulation and analytical results where the server is assumed to be Markovian. A simulation run consists of 5000 elementary tests which are subdivided into 10 part-tests. The simulation results are depicted with their 95% confidence intervals.

In this diagram which shows the trajectory of the time-dependent system response to a triangular overload traffic pattern where the hysteresis characteristic of the call completion rate of the switching system can clearly be recognized. This effect is caused by the state-dependency of the batch arrival process and the waiting-time dependency of the call completion probability. In order to investigate such effects, instationary performance analysis is indispensable. Therefore, the instationary simulation technique is a useful means for studies concerning the overload phenomenon and the overload control efficiency of switching systems.

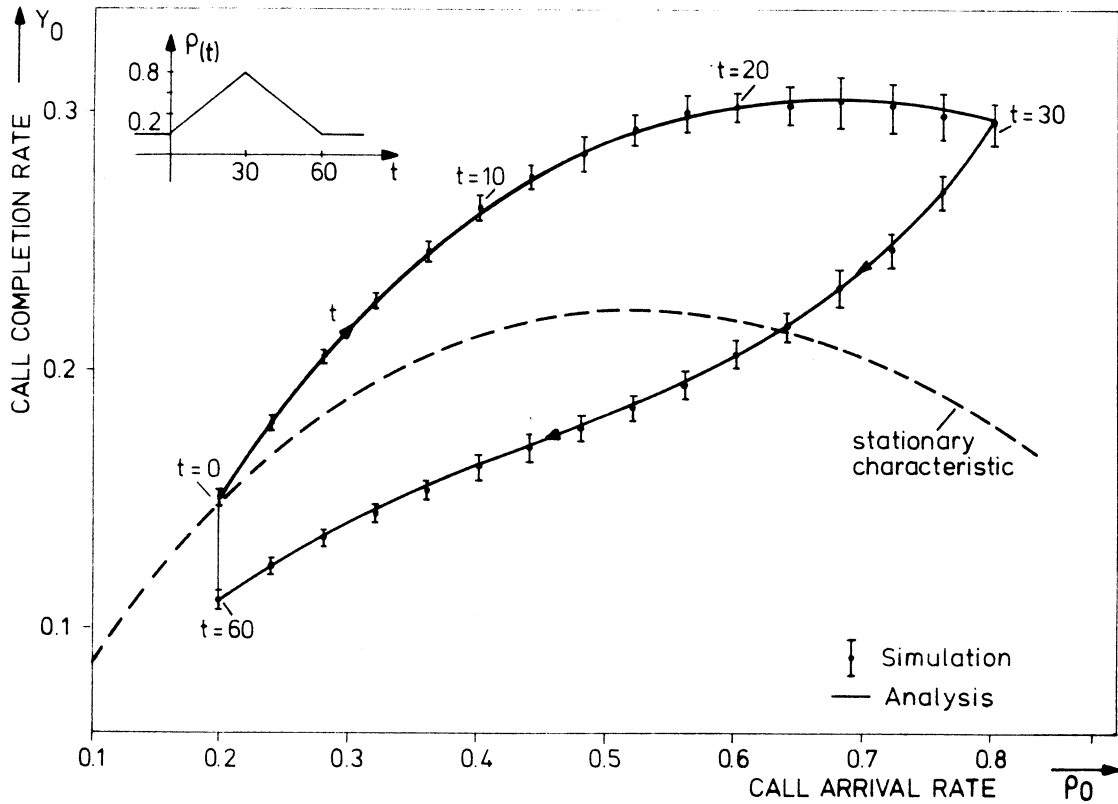


Fig.3.1 Trajectory of time-dependent system response to a triangular overload traffic pattern.

3.2 Transient Behaviour of the M(t)/G/n Queueing System

The multi-server queueing system M(t)/G/n with infinite waiting capacity and time-dependent Poissonian input process is considered. The time axis is normalized by the mean service time h and the traffic intensity is given as $\rho(t) = \lambda(t) \cdot h/n$, where n denotes the number of servers. Again, a simulation run is composed of 5000 elementary tests which are subdivided into 10 part-tests. The confidence intervals are small (<3%) and are not given in the diagrams. For the Markovian server case, simulation results are compared with numerically obtained values where good agreements are recognized. The transient responses of the M(t)/G/n (n=10) queueing system to a rectangular traffic pattern are depicted in Fig.3.2 where different service time distribution functions are considered.

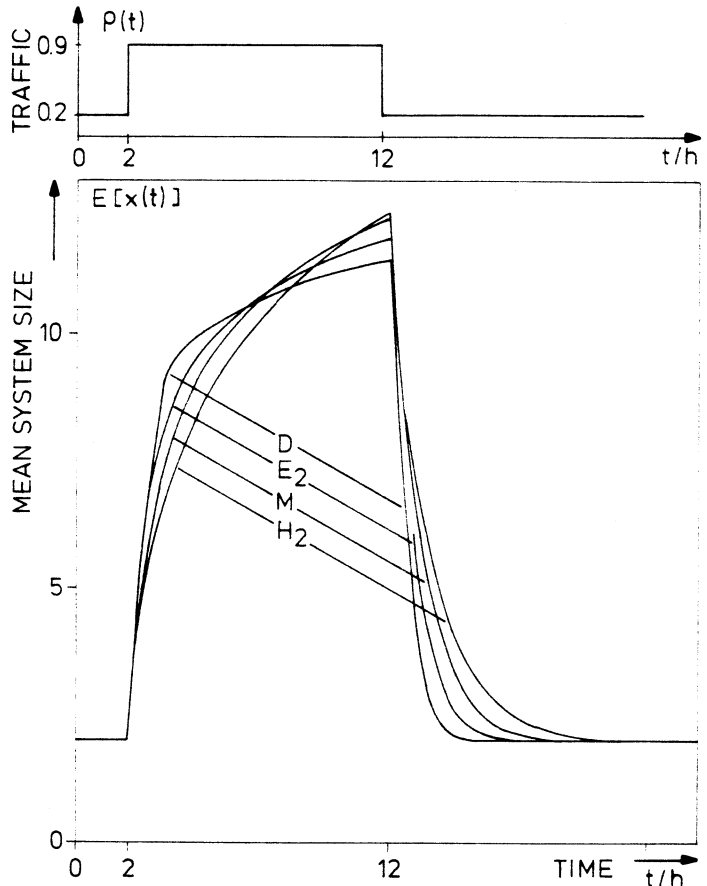


Fig.3.2 Transient responses of the M(t)/G/n queueing system to a rectangular traffic pattern (n=10).

A periodical triangular traffic pattern is considered in Fig.3.3. Starting with an initial traffic intensity $\rho_0=0.2$, where the system is in stationary conditions, the transition of the system $M(t)/G/n$ ($n=5$) into quasi-stationary conditions is shown in this diagram for different service time distribution functions. Under the quasi-stationary conditions, the system response can completely be described by observing one period of the quasi-stationary process.

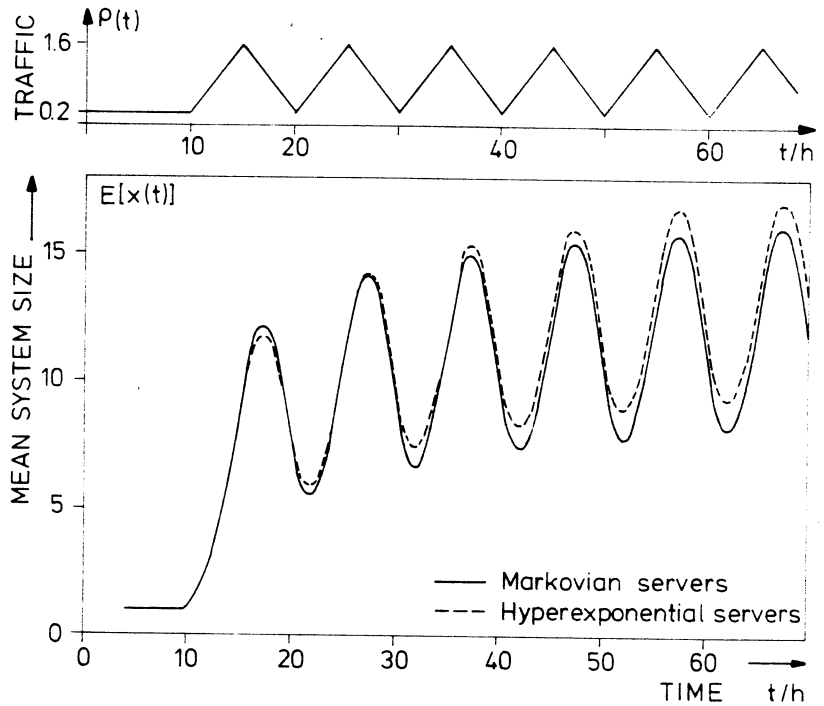


Fig.3.3 Transient responses of the $M(t)/G/n$ queueing system to a quasi-stationary, triangular traffic pattern ($n=5$).

4. CONCLUSION AND OUTLOOK

The simulation technique for instationary Poisson traffic processes forms a powerful means for performance investigations of dynamical, time-dependent behaviour of computer and communications systems, especially for overload situations in such systems. The exact simulation technique for queueing systems with generalized, time-dependent Poisson inputs can be used to validate analytical investigations. Furthermore, the technique enables system studies where analytical methods are not available.

The instationary simulation technique described here can be extended for time-dependent general input processes for which a renewal approximation is considered.

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