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Notker Gerlich

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* *Institute of Computer Science, University of Würzburg*
Am Hubland, D-97074 Würzburg, FRG
Tel.: +49 931 888-5513
E-mail: gerlich@informatik.uni-wuerzburg.de

ON THE SPATIAL MULTIPLEXING GAIN OF SDMA FOR WIRELESS LOCAL LOOP ACCESS*

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The high teletraffic demand of local loop customers as compared to cellular mobile customers requires a cellular system to have a higher capacity if used for wireless local loop access. To cope with this fact Space Division Multiple Access is proposed to be implemented on top of a cellular F/TDMA system. SDMA allows the spatial user distribution to be exploited for multiple access. Thus, the properties of the user distribution are crucial for the resulting multiplexing gain. In this paper we model the user distribution in a cell by a planar point process and derive measures of the spatial multiplexing gain.

1 Introduction

Local loop access in the future will often be wireless. It is deployment cost that will foster this development. The deployment of a wireless local loop system is cheaper and faster than the deployment of wire-line local loop with all its earth work to be carried out. New network operators, whether in many industrial countries after the fall of former telecommuncation monopolies or in fast developing countries, require effective local loop systems that are fast to deploy.

The high tele-traffic demand of local loop customers as compared to cellular mobile customers requires a cellular system to have a higher capacity when used for wireless local loop acces. A means to cope with this fact is the deployment of microcells (Lee 1991; Sarnecki et al. 1993). An alternative is to use SDMA for multiple access.

Space Division Multiple Access (SDMA) is a new multiple access scheme for cellular systems operating on top of a traditional multiplex scheme, say F/TDMA, which promises an increase of a cell's capacity. Providing the base station of a cell with an adaptive array antenna (Hudson 1981; Gabriel 1992) allows the spatial dimension of the cell to be exploited for multiple access. In F/TDMA parallelism of channels is established in frequency and time domain, whereas SDMA achieves parallelism of channels in the space domain. The adaptive array antenna's ability to transmit and receive electro-magnetic energy direction-selectively facilitates multiple radio beams to be formed and aimed towards individual users using the same frequency/timeslot. Thus, multiple users can share the same radio channel if beam patterns can be formed to guarantee a minimum carrier-to-interference ratio (C/I) for each user. Means to achieve this are beam forming and co-channel nulling (Weis 1994; Farsakh and Nossek 1994). Moreover, one has to implement algorithms to estimate the users' direction and, in mobile cellular, to track the users' movements.

In the literature approaches have been discussed to indroduce SDMA in existing cellular environments (Swales et al. 1990; Rheinschmitt et al. 1993; Tangemann et al. 1994), particularly GSM (Forssén et al. 1994; Tangemann and Rheinschmitt 1994; Bursztejn and Lakmaker 1995),

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IS-54 (Winters 1993), PCS (Goldburg and Roy 1994) and third generation systems (Tsoulos et al. 1994). Apart from providing an additional multiplexing gain SDMA exhibits additional benefits (Tangemann and Rheinschmitt 1994) like less multi-path spreading, less interference, and higher range with less power consumption. As it improves the interference situation SDMA is also discussed in connection with CDMA systems (Naguib et al. 1994), where capacity is interference limited.

In this paper we will concentrate on the spatial multiplexing gain of SDMA on top of an F/TDMA system for wireless local loop access. Here the crucial factor is the spatial distribution of the users. In order to serve a pair of users by parallel beams using the same radio channel their angular positions as seen from the base station must differ enough to be separated such that the C/I is guaranteed. For example, if all users dwell in roughly the same direction as seen from the base station spatial multiplexing is not possible. On the other hand, the multiplexing gain will be maximal if the users' directions are uniformly distributed.

Thus, we need to model the user population based on a small set of characterizing parameters. The analysis of this model together with a radio model leads to measures of the multiplexing gain.

The paper is organised as follows. In Section 2 the radio model and the user distribution models are presented. We analyse these models to derive the fresh call blocking probability and, hence, the capacity of the system in Section 3. Results of this analysis are presented in Section 4. Section 5 concludes the paper.

2 Radio and User Distribution Models

2.1 Underlying Radio Model

We start with the description of the radio model. In (Gerlich and Tangemann 1995) the "brickwall"–approximation (cf. Figure **1**) was applied to lead to the following two conditions. Between any two users sharing the same traffic channel:

(1) A minimum angular distance ϕ must be guaranteed:

$$\phi \quad \geq \quad \frac{\alpha}{2} + \frac{\beta}{2}, \tag{1}$$

where β is the angular beamwidth (cf. Figure **1**). Multipath spreading is assumed to be generated by local scatterers in a circular area around the MS with a diameter seen under angle α.

(2) The difference between the received signal strength must not exceed an upper limit. Assuming an exponential power law this translates to:

$$\frac{d_{\max}}{d_{\min}} \quad \leq \quad \Delta, \tag{2}$$

where $d_{\max}$ ($d_{\min}$) denotes the distance of the MS furthest from (nearest to) the basestation. The limit Δ is a function of D in Figure **1**.

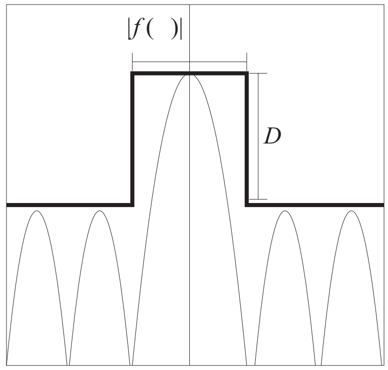

Figure **1**: Beam pattern approximation

Recent findings and the scenario we consider lead to further simplification. Fuhl and Bonek (1996) state that “[s]mall angular spreads dominate” and give the 10%-quantile of α by 4° for a realistic DCS 1800 scenario. Thus, for practical purposes it is safe to relax Eqn. 1 to read

$$\phi \geq \frac{\beta}{2}. \tag{3}$$

In the following we consider the scenario depicted in Figure **2**. The cell is a circular area of radius R served by a base station located in the center. There are no users within the circle with radius r around the base station. This modelling is justified by the fact that SDMA is considered for macro-cell coverage where base stations usually are positionend at higher positions like hills while users dwell in the vallies. If we let $R/r \leq \Delta$ then condition 2 always holds. Power control also relaxes this constraint. With ideal power control constraint 2 completely vanishes. Thus, in the follwing we will consider the relaxed first condition only.

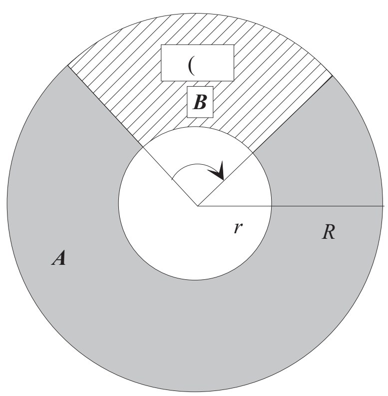

Figure **2**: Cell model

2.2 User Distribution Model

As already mentioned we aim at a stochastic description of the user population that facilitates characterization by a small set of parameters. A possible means to do so is presented by models provided by *stochastic geometry* (Stoyan et al. 1987; Cressie 1991).

Models and methods of stochastic geometry have been applied in the context of mobile communication system analysis only recently. In (Baccelli et al. 1996) a new approach to strategic communication network planning based on stochastic geometry models is proposed. Homogeneous Poisson models are applied for stochastic modelling of the subscribers and network components. It is shown how performance evaluation and optimization problems can be put down to calculating the moments of the underlying point processes. In (Tran-Gia and Gerlich 1996) the impact of user clusters on the Quality-Of-Service of a cellular mobile network is investigated. To this end the user population of a cell is modelled by planar point process models known from stochastic geometry. Latouche and Ramaswami (1996) propose a new type of planar point process model applicable to performance evaluation of mobile communication systems.

For our analysis we model the user population of a cell by realizations of stochastic planar point processes. We assume these processes to be independent and Poisson. The appealing point of the Poisson distribution is its simplicity. In the following we summarize some properties of the (in)homogeneous Poisson process. A comprehensive treatment can be found in (Cressie 1991).

In general a (planar) point process N is a random variable, which takes random choices of mappings $\mathbf{B} \mapsto N(\mathbf{B})$, where $\mathbf{B}$ is a Borel set (in the 2-dimensional plane) and $N(\mathbf{B})$ is a locally finite counting measure, the number of simple points contained in $\mathbf{B}$. A realization of N is called a point pattern.* As a random variable a point process induces a probability measure, the *distribution* of the point process N.

The distribution of the *(inhomogeneous) Poisson process* is characterized by two properties:

(1) For any collection of disjoint Borel sets $\mathbf{B}_1, \dots, \mathbf{B}_k$ the random variables $N(\mathbf{B}_1), \dots, N(\mathbf{B}_k)$ are independent.

(2) The random number $N(\mathbf{B})$ of points contained in the finite Borel set $\mathbf{B}$ has a Poisson distribution with parameter $\Lambda(\mathbf{B})$.

$$P\{N(\mathbf{B}) = n\} = \frac{\Lambda(\mathbf{B})^n}{n!} \exp[-\Lambda(\mathbf{B})] \tag{4}$$

The measure $\Lambda(\mathbf{B})$ is called the *intesity measure* of the process.

Let $\nu(\mathbf{B})$ denote the Lebesgue measure of $\mathbf{B}$, i.e. the area of $\mathbf{B}$ if $\mathbf{B}$ is regular enough. If

$$\Lambda(\mathbf{B}) = \lambda \nu(\mathbf{B}) \tag{5}$$

i.e., $\Lambda(\mathbf{B})$ is independent of the location of $\mathbf{B}$, then the Poisson process is called *homogeneous.* The parameter λ is called the *intensity* of the process and can be interpreted as the mean density of points.

*It should be noted that in this context the term "process" is somewhat misleading since it does imply a dynamic evolution in time, the notion of which is not present here.

3 Analysis

In the following we will derive the fresh call blocking probability P_B. First, we will model the user population by a homogeneous Poisson process. To investigate the impact of a non-uniform user distribution we will proceed to an inhomogenous Poisson process model in the second subsection.

Let c denote the number of F/TDMA channels and b denote the maximum number of parallel beams. Thus, the maximum number of SDMA channels is $s = cb$.

When will a fresh call (test user) be blocked? Obviously, this will be the case if all s SDMA channels are busy. Otherwise, a fresh call is blocked if the base station cannot provide a channel due to violation of the remaining relaxed condition 1 of the radio model. To check this condition we have to look at the neighbours of the test user within an angular distance of $\beta/2$ to the right and to the left. If all c F/TDMA channels are allocated to neighbours within this wedge of angular width β then the test user must be blocked. The worst case occurs if there are c neighbours each of which occupies its own F/TDMA channel. Depending on the channel allocation scheme the worst case may be encountered quite often. For minimal interference it makes perfectly sense to allocate different F/TDMA channels to users near to each other.

Expressed in terms of the planar point process these conditions translate to (cf. Figure **2**):

$$\begin{aligned} P_B &= P\{N(\mathbf{A}) \geq s \;\cup\; N(\mathbf{B}) \geq c\} \\ &= P\{N(\mathbf{A}) \geq s\} + P\{N(\mathbf{B}) \geq c\} - P\{N(\mathbf{A}) \geq s \;\cap\; N(\mathbf{B}) \geq c\} \end{aligned} \tag{6}$$

where **A** denotes the ring shaped cell and **B** denotes the "wedge" of angular width β around the test customer.

For completeness note that for calculating the number of neighbours of a point we have to use the reduced Palm distibution of the point process. The *reduced Palm distribution* (Cressie 1991) of point process N with respect to point $\mathbf{x}$ is the conditional distribution of N not counting $\mathbf{x}$ given $\mathbf{x}$ is a point of N. It is another appealing feature of the Poisson point process that the reduced Palm distribution and the distribution of the point process are identical Poisson.

Note further that Eqn. 6 overestimates the blocking probability thus leaving us on the safe side.

For ease of notation we denote the Lebesgue measure of a "wedge" of angular width γ by

$$\nu(\gamma) = \frac{\gamma}{2}(R^2 - r^2) \tag{7}$$

3.1 Homogeneous Poisson Process

Since the intensity measure of the process is independent of the location, the probability of the test user to pop up at a certain location is the same for all locations. Thus we can derive the blocking probability from a representative situation like the one depicted in Figure **2**. Bearing in mind that the number of points in a set is Poisson distributed we get straightforward:

$$P\{N(\mathbf{A}) \geq s\} = 1 - \sum_{i=0}^{s-1} \frac{[\lambda\nu(2\pi)]^i}{i!} \exp[-\lambda\nu(2\pi)] \tag{8}$$

$$P\{N(\mathbf{B}) \geq c\} = 1 - \sum_{i=0}^{c-1} \frac{[\lambda\nu(\beta)]^i}{i!} \exp[-\lambda\nu(\beta)] \tag{9}$$

For the third term of Eqn. 6 we derive by noting that $\mathbf{B} \subset \mathbf{A}$:

$$\begin{aligned} &P\{N(\mathbf{A}) \geq s \,\cap\, N(\mathbf{B}) \geq c\} \\ &= \sum_{i=c}^{\infty} P\{N(\mathbf{A}) \geq s \,\cap\, N(\mathbf{B}) = i\} \\ &= \sum_{i=c}^{\infty} P\{N(\mathbf{A} \setminus \mathbf{B}) \geq s - i \,\cap\, N(\mathbf{B}) = i\} \end{aligned}$$

Now we can exploit the fact that sets $\mathbf{A} \setminus \mathbf{B}$ and $\mathbf{B}$ are disjoint and, hence, the corresponding random variables are independent to derive:

$$\begin{aligned} &P\{N(\mathbf{A}) \geq s \,\cap\, N(\mathbf{B}) \geq c\} \\ &= \sum_{i=c}^{s-1} \sum_{j=s-i}^{\infty} P\{N(\mathbf{A} \setminus \mathbf{B}) \geq j\} \cdot P\{N(\mathbf{B}) = i\} \;+\; \sum_{i=s}^{\infty} P\{N(\mathbf{B}) = i\} \\ &= \sum_{i=c}^{s-1} P\{N(\mathbf{B}) = i\} \cdot \left[1 - \sum_{j=0}^{s-i-1} P\{N(\mathbf{A} \setminus \mathbf{B}) = j\}\right] \;+\; 1 - \sum_{i=0}^{s-1} P\{N(\mathbf{B}) = i\} \\ &= \sum_{i=c}^{s-1} \frac{[\lambda\nu(\beta)]^i}{i!} \exp[-\lambda\nu(\beta)] \cdot \left[1 - \sum_{j=0}^{s-i-1} \frac{[\lambda\nu(2\pi - \beta)]^j}{j!} \exp[-\lambda\nu(2\pi - \beta)]\right] \\ &\quad + 1 - \sum_{i=0}^{s-1} \frac{[\lambda\nu(\beta)]^i}{i!} \exp[-\lambda\nu(\beta)] \qquad (10) \end{aligned}$$

Substituting Eqns. 8 – 10 into Eqn. 6 we get the estimate of the blocking probability.

3.2 Inhomogeneous Poisson Process

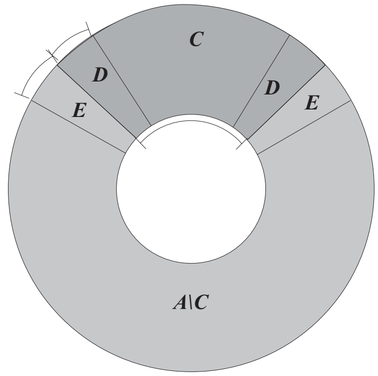

Figure **3**: Inhomogeneous case

In order to investigate the impact of a non-uniform user distribution we consider the following simple inhomogeneous Poisson point process (cf. Figure **3**). Again, we have the ring shaped cell as above. On the wedge $\mathbf{C}$ of angular width γ we have a homogeneous Poisson process with intensity λ_0. On the remaining $\mathbf{A} \setminus \mathbf{C}$ of the cell we have a homogeneous Poisson process with intensity λ_1. Thus the intensity measure reads

$$\Lambda_\gamma(\mathbf{B}) \;=\; \lambda_0\nu(\mathbf{B} \cap \mathbf{C}) \;+\; \lambda_1\nu(\mathbf{B} \cap (\mathbf{A} \setminus \mathbf{C})) \qquad (11)$$

Since the distribution of the sum of two Poisson random variables is also Poisson the process is a inhomogenous Poisson process as defined above. Without loss of generality we let $\lambda_0 > \lambda_1$ and $\gamma \geq \beta$.

Due to the intensity measure depending on the location the probability for the test user to show up is higher for locations within $\mathbf{C}$ than for locations within $\mathbf{A} \setminus \mathbf{C}$. Additionally, we have to take care of cases where a part of the wedge of angular width β around the test customer falls into $\mathbf{C}$. Consequently, we have to deal with four cases (cf. Figure **3**): The test user pops up in

(1) $\mathbf{C} \setminus \mathbf{D}$

(2) $\mathbf{A} \setminus (\mathbf{C} \cup \mathbf{E})$

(3) $\mathbf{D}$

(4) $\mathbf{E}$

For each of the cases $i = 1, \ldots, 4$ we will derive the probability of occurence $p_{(i)}$ and the blocking probability $P_B^{(i)}$. The overall blocking probability arises from applying the complete probability formula:

$$P_B = \sum_{i=1}^{4} P_B^{(i)} p_{(i)} \tag{12}$$

Let us derive the probability of the test customer $\mathbf{x}$ to show up within some set $\mathbf{B}$. Given there are i points in set $\mathbf{B}$ and j points in set $\mathbf{A} \setminus \mathbf{B}$ the probability to choose a point of $\mathbf{B}$ is $i/(i+j)$. Thus, since $\mathbf{B}$ and $\mathbf{A} \setminus \mathbf{B}$ are disjoint

$$\begin{aligned} P\{\mathbf{x} \in \mathbf{B}\} &= \sum_{i=1}^{\infty} \sum_{j=0}^{\infty} \frac{i}{i+j} P\{N(\mathbf{A} \setminus \mathbf{B}) = j\}\, P\{N(\mathbf{B}) = i\} \\ &= \exp[-\Lambda_\gamma(\mathbf{A} \setminus \mathbf{B}) - \Lambda_\gamma(\mathbf{B})] \sum_{i=1}^{\infty} \sum_{j=0}^{\infty} \frac{i \Lambda_\gamma(\mathbf{A} \setminus \mathbf{B})^j\, \Lambda_\gamma(\mathbf{B})^i}{(i+j) j! i!} \end{aligned} \tag{13}$$

Note, that applying Eqn. 11 the argument of the exponential turns out to be constant (only depending on γ). Thus, we have

$$\begin{aligned} p_{(1)} &= P\{\mathbf{x} \in \mathbf{C} \setminus \mathbf{D}\} & p_{(3)} &= P\{\mathbf{x} \in \mathbf{D}\} \\ p_{(2)} &= P\{\mathbf{x} \in \mathbf{A} \setminus (\mathbf{C} \cup \mathbf{E})\} & p_{(4)} &= P\{\mathbf{x} \in \mathbf{E}\} \end{aligned} \tag{14}$$

For deriving the blocking probability for each case we again start with Eqn. 6. Now Eqns. 8 – 10 read

$$\begin{aligned} P\{N(\mathbf{A}) \geq s\} &= 1 - \sum_{i=0}^{s-1} \frac{[\Lambda_\gamma(\mathbf{A})]^i}{i!} \exp[-\Lambda_\gamma(\mathbf{A})] \\ &= 1 - \sum_{i=0}^{s-1} \frac{[\lambda_0 \nu(\gamma) + \lambda_1 \nu(2\pi - \gamma)]^i}{i!} \exp[-\lambda_0 \nu(\gamma) - \lambda_1 \nu(2\pi - \gamma)] \end{aligned} \tag{15}$$

$$P\{N(\mathbf{B}) \geq c\} = 1 - \sum_{i=0}^{c-1} \frac{[\Lambda_\gamma(\mathbf{B})]^i}{i!} \exp[-\Lambda_\gamma(\mathbf{B})] \tag{16}$$

$$
\begin{aligned}
&P\{N(\mathbf{A}) \geq s \ \cap \ N(\mathbf{B}) \geq c\} \\
&= \sum_{i=c}^{s-1} \frac{[\Lambda_\gamma(\mathbf{B})]^i}{i!} \exp[-\Lambda_\gamma(\mathbf{B})] \cdot \left[1 - \sum_{j=0}^{s-i-1} \frac{[\Lambda_\gamma(\mathbf{A} \setminus \mathbf{B})]^j}{j!} \exp[-\Lambda_\gamma(\mathbf{A} \setminus \mathbf{B})]\right] \\
&\quad + 1 - \sum_{i=0}^{s-1} \frac{[\Lambda_\gamma(\mathbf{B})]^i}{i!} \exp[-\Lambda_\gamma(\mathbf{B})] \qquad (17)
\end{aligned}
$$

For further development of Eqns. 16 and 17 we have to distinguish the four cases:

(1) If test user $\mathbf{x} \in \mathbf{C} \setminus \mathbf{D}$ then $\mathbf{B}$ is completely embedded in $\mathbf{C}$. $\mathbf{A} \setminus \mathbf{B}$ has two parts: one of angular width $\gamma - \beta$ in $\mathbf{C}$ where the intensity is λ_0 and the second of angular width $2\pi - \gamma$ in $\mathbf{A} \setminus C$, where the intensity is λ_0. Thus, we have

$$\Lambda_\gamma(\mathbf{B}) = \lambda_0 \nu(\beta), \qquad (18)$$

$$\Lambda_\gamma(\mathbf{A} \setminus \mathbf{B}) = \lambda_0 \nu(\gamma - \beta) + \lambda_1 \nu(2\pi - \gamma). \qquad (19)$$

(2) If test user $\mathbf{x} \in \mathbf{A} \setminus (\mathbf{C} \cup \mathbf{E})$ then $\mathbf{B}$ is completely embedded in $\mathbf{A} \setminus \mathbf{C}$. Thus, analogously to (1) we have

$$\Lambda_\gamma(\mathbf{B}) = \lambda_1 \nu(\beta), \qquad (20)$$

$$\Lambda_\gamma(\mathbf{A} \setminus \mathbf{B}) = \lambda_0 \nu(2\pi - \gamma - \beta) + \lambda_1 \nu(\gamma). \qquad (21)$$

(3) If test user $\mathbf{x} \in \mathbf{D}$ then $\mathbf{B}$ has two parts: one in $\mathbf{C}$ and another in $\mathbf{A} \setminus \mathbf{C}$. Denoting the angular width of the part in $\mathbf{A} \setminus \mathbf{C}$ by ω we get:

$$\Lambda_\gamma(\mathbf{B}) = \lambda_0 \nu(\beta - \omega) + \lambda_1 \nu(\omega), \qquad (22)$$

$$\Lambda_\gamma(\mathbf{A} \setminus \mathbf{B}) = \lambda_0 \nu(\gamma - \beta + \omega) + \lambda_1 \nu(2\pi - \gamma - \omega). \qquad (23)$$

Now, ω is the realization of the uniformly distributed random variable $\Omega \in [0, \beta/2]$. Due to

$$P\{\Omega \leq \omega\} = \frac{\omega}{\beta/2} \qquad (24)$$

unconditioning with respect to ω gives rise to

$$\Lambda_\gamma(\mathbf{B}) = \frac{2}{\beta} \int_{\omega=0}^{\beta/2} \lambda_0 \nu(\beta - \omega) + \lambda_1 \nu(\omega)\, d\omega, \qquad (25)$$

$$\Lambda_\gamma(\mathbf{A} \setminus \mathbf{B}) = \frac{2}{\beta} \int_{\omega=0}^{\beta/2} \lambda_0 \nu(\gamma - \beta + \omega) + \lambda_1 \nu(2\pi - \gamma - \omega)\, d\omega. \qquad (26)$$

(4) Analogous to (3) we have:

$$\Lambda_\gamma(\mathbf{B}) = \frac{2}{\beta}\int_{\omega=0}^{\beta/2} \lambda_0\nu(\omega) + \lambda_1\nu(\beta-\omega)\,d\omega, \tag{27}$$

$$\Lambda_\gamma(\mathbf{A}\setminus\mathbf{B}) = \frac{2}{\beta}\int_{\omega=0}^{\beta/2} \lambda_0\nu(\gamma-\omega) + \lambda_1\nu(2\pi-\gamma-\beta+\omega)\,d\omega. \tag{28}$$

Reviewing Eqn. 7 we see that the integrals can explicitly be solved.

4 Results

In the following we will present results for a DCS 1800 scenario like in (Fuhl and Molisch 1996): $r = 16\ m$, $R = 3.2\ km$, $c = 7$ (i.e. one carrier with 8 time slots, one slot reserved for control channels), $b = 4$ (i.e. $M = 8$ antenna elements (Tangemann et al. 1994)).

In Figure **4** the resulting blocking probability is depicted versus the intensity λ in the homogeneous case. The solid curves are results for $\beta \in \{15°, 30°, 45°\}$. Additionaly, we choose β from a Normal distribution $\mathcal{N}(\beta, \sigma)$ with standard deviation $\sigma = 2.5°$ and means $\beta \in \{15°, 30°, 45°\}$ (dashed curves). The results were calculated using the formulae of Section 3 and the complete probability formula. Doing so, we take into account the effect of the scattering circle which we neclected in deriving the formulae of Section 3. Fuhl and Molisch (1996) give the 5% quantile of α by $\alpha = 5°$ for the scenario considered. Since the 5% quantile of a normal distribution is given by $\beta + 2\sigma$, we have $2\sigma = 5°$. Table **1** shows the resulting spectrum efficiency gain

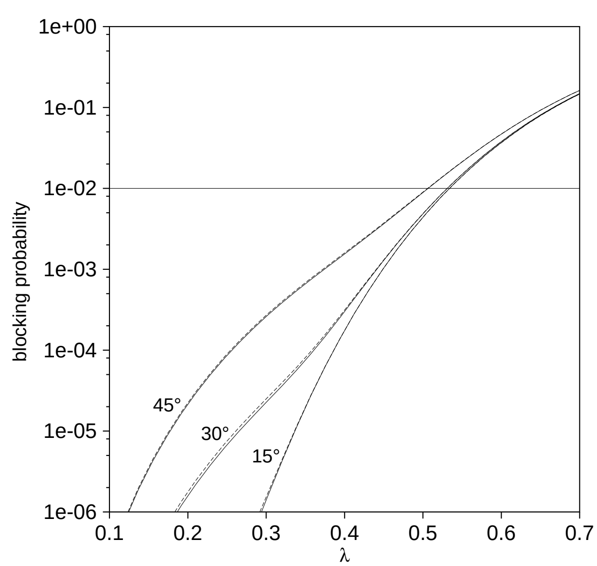

Figure **4**: Blocking Probability vs. λ

E, which is defined as the ratio of the spectrum efficiency (number of users per MHz and km^2) of the SDMA system and the plain F/TDMA system, under the constraint $P_B \leq 10^{-2}$. Thus, E expresses the capacity gain of SDMA as compared to the plain F/TDMA system. Since both systems are covering the same area while using the same amount of spectrum E is given by the ratio of the mean number of users that can be served by either system. Since we assume the user distribution to be Poisson this means that the maximum mean number of active users in the cell to guarantee a blocking probability of $P_B = 10^{-2}$ of the 7 channels F/TDMA system is 2.33.

Table **1**: Spectrum Efficiency Gain for $P_B = 10^{-2}$, Homogeneous Case

β	E
$15°$	7.37
$\sim \mathcal{N}(15°, 2.5°)$	7.37
$30°$	7.33
$\sim \mathcal{N}(30°, 2.5°)$	7.33
$45°$	6.99
$\sim \mathcal{N}(45°, 2.5°)$	6.98

First of all we note that the angular spread as accounted for by the normal distributed β has no visible effect on the blocking probability and the spectrum efficiency gain. The influence of the angular beamwidth β also is rather small with respect to blocking probability and spectrum efficiency gain. For a blocking probability of $P_B = 10^{-2}$ there is only a small difference in the spectrum efficiency gain when doubling the beamwidth and even tripling affects the gain only slightly.

Table **2** shows results for the inhomogenous scenario with $\lambda_0 = 2\lambda_1$ for $\gamma \in \{15°, 30°, 45°, 60°\}$ while Table **3** shows the results for $\lambda_0 = 10\lambda_1$ for $\gamma = 60°$. Since we had $\gamma \geq \beta$ some of the places in the table remain empty.

Table **2**: Spectrum Efficiency Gain for $P_B = 10^{-2}$, $\lambda_0 = 2\lambda_1$

β	E			
	$\gamma = 15°$	$30°$	$45°$	$60°$
$15°$	7.37	7.37	7.37	7.37
$30°$		7.35	7.29	7.25
$45°$			7.18	6.84

Table **3**: Spectrum Efficiency Gain for $P_B = 10^{-2}$, $\lambda_0 = 10\lambda_1$, $\gamma = 60°$

β	E
$15°$	6.52
$30°$	3.62
$45°$	2.63

The impact of the inhomogenity is only slightly felt with respect to the spectrum efficiency gain, if the ratio of the both intensities is moderate. If β is small then there is almost no effect. Regarding the effect of β for fixed γ the results exhibited the same behaviour as in the homogeneous case. Comparing the results of the inhomogeneous case with the results of the homogeneous case we find identity, if β is small. For $\beta \in \{30°, 45°\}$ we find a slightly larger spectrum efficiency gain as compared to the homogeneous case, if γ is small.

The picture changes, if the difference of both intensities grows as was expected. Now β becomes important. With a ratio of $\lambda_1/\lambda_0 = 10$ for $\gamma = 60°$ the spectrum efficiency is almost devided by three when β is tripled.

5 Conclusion and Outlook

The aim of this paper was to investigate for the impact of the user distribution on the multiplexing gain of an SDMA system. To this end we modelled the user distribution of a cell by a (in)homogeneous Poisson spatial point process. For both models we derived formulae for estimates of the blocking probability.

The results indicate that the spectrum efficiciency gain is quite robust against an increasing angular beamwidth as long as the user distribution is quite uniform. The beamwidth becomes important only if the user distribution is inhomogeneous. In this case a small beamwidth is desired. The effect of angular spread caused by local scatterers is neclectable.

We further conclude that spatial point processes can be used to model the distribution of a cellular mobile system. The appealing feature of the Poisson spatial point process is that its properties are dependent on only a small set of parameters. Drawbacks are the fact that the formulae get quite involved even for simple models and the fact that time is not modelled by this type of processes. One may be able to use space-time point processes but this will lead to even more complex formulae.

Nevertheless, spatial point processes may prove useful when dimensioning a system. Dimensioning requires a measure that calculates the number of beams that are feasible given a particular characterization of the user distribution. Thus, the measure sought for expresses the inherent potential for spatial parallelism of the user distribution. In the following we will propose a candidate.

Let $\Phi = \{p_1, \ldots, p_n\}$ a realization of the planar point process that describes the user distribution (see (Cressie 1991) for methods to generate realizations of point processes). Further, write $p_i \leftrightarrow p_j$ if (p_i, p_j) meets the parallelism conditions of Section 2. Define a clique of size i by $\phi_i = \{p_{j,1}, \ldots, p_{j,i} \ : \ p_{j,k} \leftrightarrow p_{j,l}\}$. Clearly, the possible number of cliques of size i is

$$c_i \ = \ |\{\phi_{i,1}, \ldots, \phi_{i,c_i}\}| \ \leq \ \binom{n}{i}. \tag{29}$$

The intended measure G must meet the following conditions: Its minimum must be the largest i that ensures all i-tupels to be served in parallel; its maximum is the largest i allowing at least one i-tupel to be served in parallel.

$$G_{\min} = \max i : c_i = \binom{n}{i}, \quad (30)$$

$$G_{\max} = \max i : c_i \neq 0. \quad (31)$$

As the truth lies somewhere inbetween we may define:

$$G = \sum_{i=1}^{n} \frac{c_i}{\binom{n}{i}}. \quad (32)$$

Clearly, both above conditions are met.

First experiments comparing the grade of parallelism as predicted by G and the grade of parallelism achieved by simulation of a city scenario are encouraging and subject to ongoing research.

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