University of Würzburg
Institute of Computer Science
Research Report Series

Steady-State Analysis of the Rate-Based Congestion Control Mechanism for ABR Services in ATM Networks

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Report No. 114 Mai 1995

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Abstract

In ATM network environments, congestion control plays an important role to support different quality of service guarantees for a large variety of traffic types. While for real-time communications, like CBR and VBR services, a preventive open-loop congestion control mechanism is applied, a reactive closed-loop mechanism has been suggested for the ABR service class, which was introduced mainly for data communications.
This paper presents an analysis to evaluate the performance of the rate-based congestion control mechanism developed by the ATM Forum. We derive formulae to compute the evolution of the allowed cell rate and the resulting buffer content during steady state. Therefore, we focus on traffic scenarios consisting of single sources as well as of homogeneous mixes of ABR sources.
From numerical examples we can conclude that an appropriate setting of the control parameters is important to achieve a reasonable performance of the rate-based control mechanism, especially to obtain low buffer requirements in conjunction with a high utilization of network resources. Furthermore, a strong dependence on the number of active connections is observed.

1 Introduction

One major advantage of the *Asynchronous Transfer Mode* (ATM) is the support of a large variety of traffic types with multiple *Quality of Service* (QoS) guarantees in WAN and LAN environments. This capability, however, can only be achieved if effective traffic management mechanisms are implemented, which facilitate an efficient and stable operation of ATM networks [3]. The most essential traffic management mechanisms are connection admission control, usage parameter control, congestion control, priority control and traffic shaping. Among these, the congestion control offers the greatest challenge to network and system designers.

For ATM networks, two different congestion control strategies have been discussed. Depending on the service type used by a connection, open-loop or closed-loop control is suggested [5].

The first one, the open-loop mechanism, is applied for CBR and VBR connections and limits each connection's usable bandwidth according to a number of source traffic descriptors negotiated at connection setup. Based on these descriptors, sufficient network resources are allocated to guarantee a certain QoS during the connection's life-time. Therefore, the open-loop strategy is also referred to as preventive congestion control.

In case of data communications, bandwidth requirements are usually not known at connection setup, which makes the open-loop strategy insufficient for this kind of service. Furthermore, the negotiated rate cannot be exceeded, not even when the network is in a low load condition. This results in an inefficient use of network resources, since for data communications, the cell transmission rate can be adjusted to the current congestion status of the network.

These reasons have lead the ATM Forum to define another service type mainly for data communications, the so-called *Available Bit Rate* (ABR) service. To support ABR connections, a closed-loop congestion control is applied, which is a reactive control mechanism which dynamically regulates the cell transmission rate of each ABR connection by using feedback information from the network. This mechanism has been developed by the Traffic Management Group of the ATM Forum and operates on an end-to-end basis. It combines features from the *Forward Explicit Congestion Notification* (FECN) and *Backward Explicit Congestion Notification* (BECN) scheme well-known from conventional packet switching networks with an explicit rate mechanism.

In this paper, we present an analytical approach to evaluate performance measures of the closed-loop congestion control mechanism proposed by the ATM Forum in the draft version of the Traffic Management Specification 4.0. This approach allows to derive appropriate settings of the control parameters, to provide a proper traffic management in ATM network environments.

The rest of this paper is organized as follows. In Section 2, the rate-based congestion control mechanism is described. An analytical approach to derive the evolution of the allowed cell transmission rates of the ABR sources and the resulting buffer content is presented in Section 3. We compute cell transmission delays and present upper and lower bounds for each measure. Section 4 discusses numerical results obtained with our analysis and the paper concludes with a brief outlook in Section 5.

2 The Closed-Loop Rate Control Mechanism

The ATM Forum traffic management specification [2] defines only the behavior of the *Source End System* (SES) and the *Destination End System* (DES). Therefore, the implementation of the switches is left to the manufacturers. In the following, the behavior of the end systems is described. We consider the network configuration given in Figure 1.

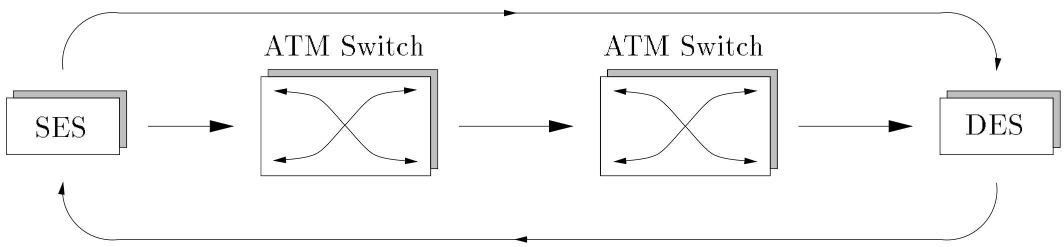

Figure 1: *Basic configuration for rate-based congestion control*

The SES, which is connected with the DES via a number of switches, is permitted to emit cells according to an *Allowed Cell Rate* (ACR) changing with the congestion status of the network. At connection setup, an *Initial Cell Rate* (ICR), a *Minimum Cell Rate* (MCR) and a *Peak Cell Rate* (PCR) are negotiated. The ACR may vary between the MCR and the PCR, and the SES is allowed to transmit cells with a rate of at most ACR.

To receive feedback information from the network, the SES sends a forward *Resource Management* (RM) cell every N_{rm} data cells. When the forward RM cell arrives at the DES, it is returned as backward RM cell to the SES. Information about the current congestion status of the network resources is provided by the switches and the DES, which may alter the content of the RM cells. At the arrival of a backward RM cell, the SES adjusts its ACR according to the *Congestion Indication* (CI) bit and the *Explicit Rate* (ER) field of the RM cell.

When a backward RM cell is received at time t with CI=1, the ACR should be reduced by $ACR(t) \cdot N_{rm}$ / RDF down to the MCR, where RDF is called *Rate Decrease Factor*. If the backward RM cell has CI=0, the ACR is increased by $AIR \cdot N_{rm}$ but not beyond the PCR. The factor AIR is called *Additive Increase Rate*. If the ER in the RM cell is lower than the current ACR, but higher than the MCR, the ACR is set to this value. To take care of RM cell failures and problems with connections having idle phases, a number of time-out mechanisms have been developed, too. Since we do not take into account such situations in this paper, a detailed description, which can be found e.g. in [2], is omitted.

The CI bit and the ER field in the RM cells are set as follows. Congestion is detected by the network according to the queue length in the switch buffers. Depending on the switch architecture, different actions are performed:

- *Explicit Forward Congestion Indication Switch (EFCI switch):*
  If the queue length is exceeding an upper threshold, the switch sets the EFCI bit in the header of data cells equal to one, until the queue length falls below a lower threshold. The DES sets the CI bit in each backward RM cell to the EFCI value of the data cell received last. The switch can optionally set the CI bit in backward RM cells to ensure that the source does not increase its rate.

- *Explicit Rate Switch (ER switch):*
  EDS switches are provided with an intelligent marking and an explicit rate setting capability. This allows to selectively reduce the rates of sources by marking the CI bit or setting the ER field according to the degree of congestion in forward and/or backward RM cells.

Furthermore, backward RM cells may be generated by both types of switches. This option is however not considered in this paper. A detailed description of possible switch behaviors is presented e.g. in [6].

3 Performance Analysis

In the following, we describe an analytical model to compute the steady-state behavior of the rate control mechanism, which allows to estimate buffer requirements and transfer delays. In this paper, we focus only on EFCI switches without CI bit setting. The analysis for ER switches will be presented in a companion paper.

The additive increase and the multiplicative decrease of the source rates are modeled using differential equations. Such an approach has been used e.g. in [6] and [7]. The control mechanisms considered in these papers differ, however, from the one developed by the ATM Forum. Furthermore, [7] focuses only on maximum values for the queue length and the source rate whereas in [6] MCR and PCR are not taken into account. Other studies on feedback mechanisms, which are not so close related to the one analyzed in this paper, can be found e.g. in [4, 8].

For the derivation of our analysis, we make use of the network model depicted in Figure 2. A number of ABR traffic sources are connected to their destinations via a single bottleneck link. The queue is assumed to be of infinite capacity and the traffic sources are in a saturated state, i.e. they have always cells to transmit. Priority mechanisms are not implemented.

We denote the *Round-Trip Time* (RTT) from the SES to the DES by τ, the propagation delay between source and the EFCI switch by τ_1, and between switch and the DES by τ_2. Thus, the feedback time τ_f is given by

$$\tau_f = 2 \cdot \tau_2 + \tau_1 \quad . \tag{1}$$

When the queue length at the EFCI switch exceeds Q_H, the switch detects congestion and marks the EFCI bit in the header of data cells. Congestion is regarded as terminated, if

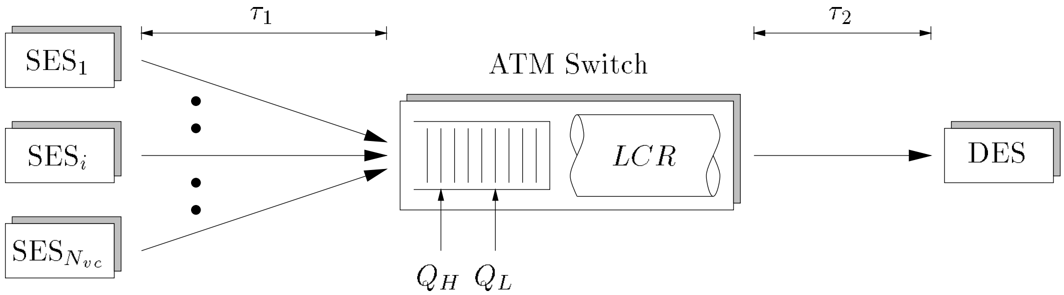

Figure 2: *Network model with a single bottleneck link*

the queue length goes below Q_L. The link is assumed to operate with a fixed transmission speed of LCR.

In the next subsection, we derive the dynamic behavior of ACR, queue length, and cell delay for a single source case. These results are extended in Section 3.2 to deal with a mix of homogeneous sources.

3.1 The Single Source Case

3.1.1 Dynamic System Behavior

The dynamic behavior of the ACR, denoted by $ACR(t)$, and the queue length $Q(t)$ can be described by a cycle consisting of four major phases, which occur periodically. An example cycle is illustrated in Figure 3.

Depending on the phase, the ACR is increased or decreased with respect to the arrival rate of backward RM cells, which depends on the number of cells currently queued up in the switch. The system behavior in each of these major phases is discussed in the following.

Phase 1

Assume that the initial values $ACR(t_0)$ and $Q(t_0 + \tau_1)$ at the beginning of Phase 1 are given. The queue is observed τ_1 time units after the ACR, due to the propagation delay between the SES and the EFCI switch. During this first phase, no congestion is detected by the switch, since the queue length $Q(t)$ is smaller than Q_H. Thus, the ACR is increased by $N_{rm} \cdot AIR$ at each arrival of a backward RM cell until the PCR is reached. These arrivals occur at a constant rate of $LCR \ / \ N_{rm}$, since the switch is fully utilized.

The additive increase in this phase can be expressed by the following differential equation

$$\frac{dACR_1(t)}{dt} = LCR \cdot AIR \quad , \tag{2}$$

which is solved by

$$ACR_1(\Delta t) = ACR(t_0) + LCR \cdot AIR \cdot \Delta t \quad , \tag{3}$$

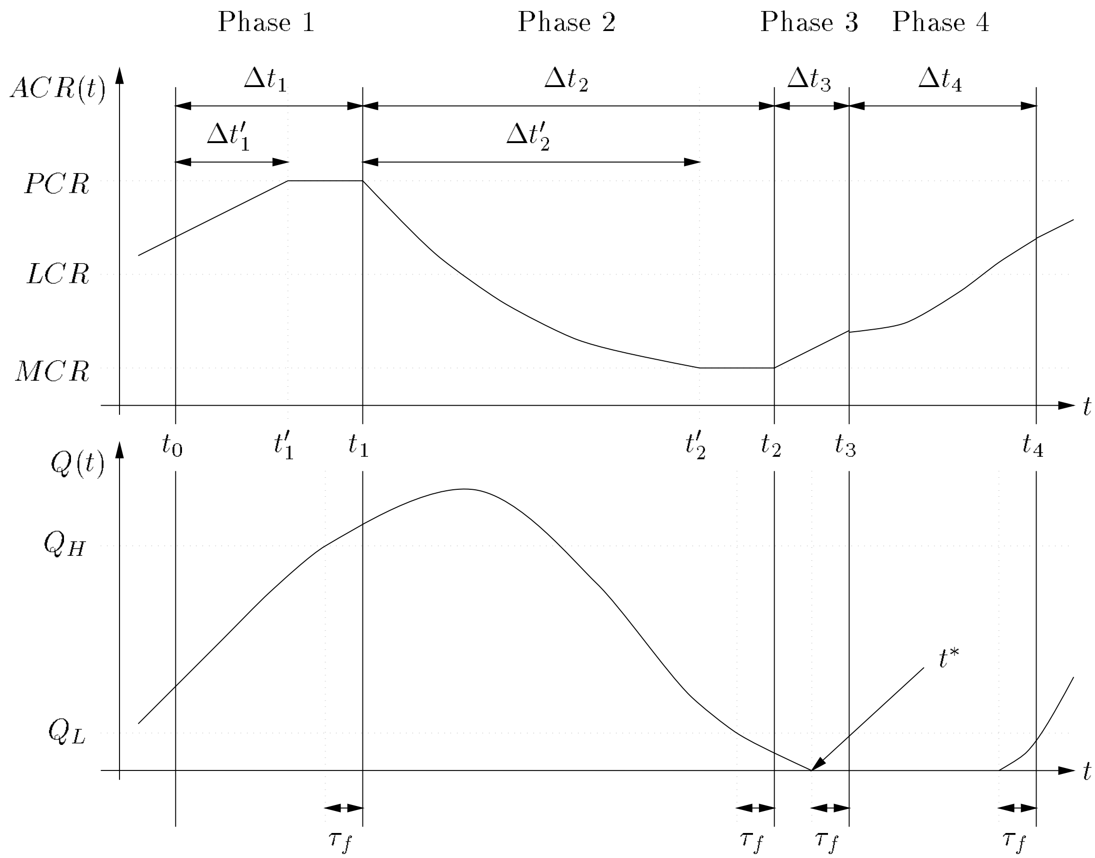

Figure 3: *Dynamic behavior of $ACR(t)$ and $Q(t)$ for EFCI switches*

with respect to the initial condition $ACR_1(0) = ACR(t_0)$. The interval length Δt is given relative to the starting time of the current phase, which is t_0 for Phase 1. Using this result, we can describe the evolution of the queue length within Phase 1 by

$$
\begin{aligned}
Q_1(\Delta t) &= Q(t_0 + \tau_1) + \int_{x=\tau_1}^{\Delta t} (ACR_1(x - \tau_1) - LCR)\, dx \\
&= Q(t_0 + \tau_1) - LCR\,(\Delta t - \tau_1) + ACR(t_0)\,(\Delta t - \tau_1) + \\
&\quad \frac{1}{2}\, LCR \cdot AIR\,(\Delta t^2 - 2\,\Delta t\,\tau_1 + \tau_1^2) \quad .
\end{aligned} \tag{4}
$$

To obtain Δt_1, which is the length of Phase 1, we have to distinguish between two cases, depending on whether the PCR of the connection is attained or not. If the ACR remains below the PCR, Δt_1 is given by the time until the queue exceeds Q_H plus the feedback time τ_f, i.e.

$$\Delta t_1 = Q_1^{-1}(Q_H) + \tau_f \quad . \tag{5}$$

The initial values for Phase 2 are then computed by

$$ACR(t_1) = ACR_1(\Delta t_1) \tag{6}$$

and

$$Q(t_1 + \tau_1) = Q_1(\Delta t_1 + \tau_1) \quad . \tag{7}$$

Otherwise, we first have to determine the time interval $\Delta t'_1$ until the PCR is attained, which is given by

$$\Delta t'_1 = ACR_1^{-1}(PCR) \quad . \tag{8}$$

From the time instant t'_1 on, the ACR remains unchanged, until congestion is detected by the switch. For the queue length at $t'_1 + \tau_1$, we get

$$Q(t'_1 + \tau_1) = Q_1(\Delta t'_1 + \tau_1) \quad . \tag{9}$$

If $Q(t'_1+\tau_1) \geq Q_H$, the length of Phase 1 is obtained by equation (6), since Q_H is exceeded before the PCR is attained. If not, the traffic source is allowed to transmit data with its PCR until congestion is detected and signaled to it by a backward RM cell. Thus, we get

$$\Delta t_1 = \Delta t'_1 + \frac{Q_H - Q(t'_1 + \tau_1)}{PCR - LCR} + \tau \quad . \tag{10}$$

In both cases, we obtain the initial ACR and the queue length for Phase 2 by

$$ACR(t_1) = PCR \quad , \tag{11}$$

$$Q(t_1 + \tau_1) = Q(t'_1 + \tau_1) + (\Delta t_1 - \Delta t'_1) \cdot (PCR - LCR) \quad . \tag{12}$$

Phase 2

Phase 2 starts when a backward RM cell indicating congestion is received by the SES and lasts until another backward RM cell indicating no congestion, i.e. the queue length in the switch is below Q_L, is received. During this time period, the ACR is decreased in a multiplicative manner at the arrival of RM cells, with a factor of N_{rm} / RDF. This is done until the MCR of the connection is attained. Like in Phase 1, the switch is fully utilized during the whole time period, and thus, backward RM cells arrive with a constant rate of LCR / N_{rm}. This decrease can be expressed by the differential equation

$$\frac{dACR_2(t)}{dt} = -ACR_2(t)\frac{LCR}{RDF} \quad , \tag{13}$$

which is solved by

$$ACR_2(\Delta t) = ACR(t_1)\, e^{-\frac{LCR}{RDF}\Delta t} \quad , \tag{14}$$

with respect to the initial condition $ACR_2(0) = ACR(t_1)$. The evolution of the queue length in Phase 2 is determined by

$$
\begin{aligned}
Q_2(\Delta t) &= Q(t_1+\tau_1) + \int\limits_{x=\tau_1}^{\Delta t} (ACR_2(x-\tau_1) - LCR)\, dx \\
&= Q(t_1+\tau_1) - LCR\,(\Delta t - \tau_1) + \\
&\quad ACR(t_1)\,\frac{RDF}{LCR}\left(1 - e^{-\frac{LCR}{RDF}(\Delta t - \tau_1)}\right) \quad .
\end{aligned} \tag{15}
$$

If the ACR does not attain the MCR, the length Δt_2 of Phase 2 is given by

$$\Delta t_2 = Q_2^{-1}(Q_L) + \tau_f \quad , \tag{16}$$

i.e. the time required until the queue length goes below Q_L plus the feedback time. The initial values for the next phase are computed as follows:

$$ACR(t_2) = ACR_2(\Delta t_2) \quad , \tag{17}$$

$$Q(t_2+\tau_1) = [Q_2(\Delta t_2 + \tau_1)]^+ \quad . \tag{18}$$

Otherwise, the MCR is approached within Phase 2 after

$$\Delta t_2' = ACR_2^{-1}(MCR) \tag{19}$$

time units, and the queue length at $t_2' + \tau_1$ is

$$Q(t_2'+\tau_1) = [Q_2(\Delta t_2' + \tau_1)]^+ \quad . \tag{20}$$

For $Q(t_2'+\tau_1) \leq Q_L$, the length of Phase 2 can be obtained using equation (16). For $Q(t_2'+\tau_1) > Q_L$, it is given by

$$\Delta t_2 = \Delta t_2' + \frac{Q_L - Q(t_2'+\tau_1)}{MCR - LCR} + \tau \quad . \tag{21}$$

In both cases, the initial values for the next phase are

$$ACR(t_2) = MCR \quad , \tag{22}$$

$$Q(t_2+\tau_1) = [Q(t_2'+\tau_1) + (\Delta t_2 - \Delta t_2') \cdot (MCR - LCR)]^+ \quad . \tag{23}$$

Now, there are two possibilities. If the queue remains non-empty until the ACR reaches LCR, we start a new cycle with Phase 1. If not, we have to continue the current cycle with Phases 3 and 4 described in the following.

Phase 3

At the beginning of Phase 3, the SES recognizes that congestion is terminated and the ACR is increased again, like in Phase 1. Thus, we have

$$ACR_3(\Delta t) = ACR(t_2) + LCR \cdot AIR \cdot \Delta t \quad , \tag{24}$$

$$\begin{aligned} Q_3(\Delta t) \quad &= \quad Q(t_2+\tau_1) - LCR\,(\Delta t - \tau_1) + ACR(t_2)(\Delta t - \tau_1) + \\ &\quad \frac{1}{2}\,LCR \cdot AIR\,(\Delta t^2 - 2\,\Delta t\,\tau_1 + \tau_1^2) \quad . \end{aligned} \tag{25}$$

However, the duration of the phase differs from that of Phase 1, since the queue is getting empty at a time t^* and thus, the arrival rate of the backward RM cells depends on the ACR of the source. In our phase model, Phase 3 lasts until this rate dependency occurs, i.e. $t_3 = t^* + \tau_f$. For brevity, we outline only the case where the PCR is not attained within this phase. Otherwise, a similar treatment as described in Phases 1 and 2 is necessary.

If $t^* \leq t_2 + \tau_1$, i.e. the queue gets empty in Phase 2, the duration Δt_3 of Phase 3 is given by

$$\Delta t_3 = \tau_f - (t_2 - t^*) \quad , \tag{26}$$

which results from the feedback delay τ_f. If this is not the case, then Δt_3 is determined by

$$\Delta t_3 = Q_3^{-1}(0) + \tau_f \quad . \tag{27}$$

The initial value for the ACR at the beginning of Phase 4 is given by

$$ACR(t_3) = ACR_3(\Delta t_3) \quad . \tag{28}$$

If $ACR(t_3)$ does not exceed LCR, then the queue remains empty at $t_3 + \tau_1$, i.e.

$$Q(t_3 + \tau_1) = 0 \quad . \tag{29}$$

If the ACR goes beyond LCR within Phase 3, we obtain the queue length by

$$Q(t_3 + \tau_1) = \frac{1}{2}\,LCR \cdot AIR\,((\Delta t_3 - \Delta t_3')^2 - 2\,(\Delta t_3 - \Delta t_3')\,\tau_1 + \tau_1^2) \quad , \tag{30}$$

where $\Delta t_3'$ is the time required to reach LCR in Phase 3:

$$\Delta t_3' = ACR_3^{-1}(LCR) \quad . \tag{31}$$

Phase 4

Now, due to the empty queue, the arrival rate of the backward RM cells at time t corresponds to the ACR of the SES τ time units before, i.e. $ACR(t-\tau)\ /\ N_{rm}$. At each RM cell arrival, the ACR is increased by $N_{rm} \cdot AIR$, which leads to the following differential equation:

$$\frac{dACR_4(t)}{dt} = AIR \cdot ACR_4(t-\tau) \quad . \tag{32}$$

With respect to $ACR_4(0) = ACR(t_3)$, equation (32) is solved by

$$ACR_4(\Delta t) = ACR(t_3)\, e^{\beta \Delta t} \quad , \tag{33}$$

where β is the root of $\beta = AIR\, e^{-\beta\tau}$. For the evolution of the queue we obtain

$$\begin{aligned} Q_4(\Delta t) &= Q(t_3+\tau_1) + \int_{x=\tau_1}^{\Delta t} \left(ACR_4(x-\tau_1) - LCR\right) dx \\ &= Q(t_3+\tau_1) - LCR\,(\Delta t - \tau_1) + \frac{1}{\beta} ACR(t_3) \left(e^{\beta(\Delta t - \tau_1)} - 1\right) \quad . \end{aligned} \tag{34}$$

For the following derivation, we again omit the case where the PCR is attained within Phase 4. If $Q(t_3+\tau_1) = 0$, which implies $ACR(t_3) < LCR$, we obtain the duration of Phase 4 by

$$\Delta t_4 = ACR_4^{-1}(LCR) + \tau \quad . \tag{35}$$

The ACR and the queue length at the beginning of the next phase, which is now Phase 1 in the next cycle, are given by

$$ACR(t_4) = ACR_4(\Delta t_4) \quad , \tag{36}$$

$$Q(t_4+\tau_1) = -LCR \cdot \tau + \frac{1}{\beta} ACR(t_3) \left(e^{\beta \Delta t_4} - e^{\beta(\Delta t_4 - \tau)}\right) \quad . \tag{37}$$

For $Q(t_3+\tau_1) > 0$, we obtain

$$\Delta t_4 = \tau - (\Delta t_3 - \Delta t_3') \quad , \tag{38}$$

where $\Delta t_3'$ is the time required to reach LCR in Phase 3 (cf. equation (31)). In this case, the initial values for the succeeding Phase 1 are computed as

$$ACR(t_4) = ACR_4(\Delta t_4) \quad , \tag{39}$$

$$Q(t_4+\tau_1) = Q_4(\Delta t_4+\tau_1) \quad . \tag{40}$$

By iterating until convergence is reached, we obtain the dynamic behavior of $ACR(t)$ and $Q(t)$ in steady state.

The evolution of the end-to-end delay between the SES and the DES experienced by a cell emitted at time t can easily be derived out of the queue length $Q(t)$. Since it takes τ_1 time units for a cell to arrive at the switch, the total end-to-end delay $D(t)$ is determined by

$$D(t) = \tau + Q(t+\tau_1)/LCR \quad . \tag{41}$$

3.1.2 Maximum and Minimum Queue Length

In this subsection, we derive exact values for the maximum and minimum queue length in each cycle. Furthermore, we present bounds for these values, which are independent of the cycle investigated. Using these bounds, buffer requirements can be estimated and the existence of periods, where the bottleneck link is underutilized can be detected.

The maximum queue length is attained within Phase 2. Let $t_{\max}$ be the time, when the ACR goes below LCR. Then, $\Delta t_{\max}$ is given by

$$\Delta t_{\max} = ACR_2^{-1}(LCR) = -\ln(LCR/ACR(t_1)) \cdot \frac{RDF}{LCR} \quad . \tag{42}$$

The maximum queue length $Q_{\max}$ in a given cycle is obtained by

$$\begin{aligned} Q_{\max} &= Q(t_{\max}+\tau_1) \\ &= Q(t_1+\tau_1) - LCR\cdot\Delta t_{\max} + ACR(t_1)\,\frac{RDF}{LCR}\left(1-e^{-\frac{LCR}{RDF}\Delta t_{\max}}\right) \quad . \end{aligned} \tag{43}$$

In the following, we derive an upper-bound for $Q_{\max}$ which is independent of the cycle number. $ACR(t_1)$ is upper bounded by

$$ACR(t_1) \leq PCR \quad . \tag{44}$$

For $Q(t_1+\tau_1)$, an upper bound is given by

$$Q(t_1+\tau_1) \leq Q_H + \int\limits_{x=\Delta t_1-\tau_f}^{\Delta t_1+\tau_1} (PCR-LCR)\,dx = Q_H + \tau\,(PCR-LCR) \tag{45}$$

Therefore, an upper bound for $Q_{\max}$ is obtained by

$$
\begin{aligned}
Q_{\max} \leq \; & Q_H + \tau\,(PCR - LCR) + \\
& RDF \cdot \ln\left(\frac{LCR}{PCR}\right) + RDF \cdot \left(\frac{PCR}{LCR} - 1\right) \quad .
\end{aligned}
\tag{46}
$$

and for the maximum delay $D_{\max}$ we get

$$
D_{\max} = \tau + Q_{\max}/LCR \quad . \tag{47}
$$

From equation (46), a strong dependence of the maximum queue length on the relation between PCR and LCR can be observed. On the other hand, the maximum queue size increases with the RTT τ, since the PCR is generally larger than LCR.

The minimum queue length $Q_{\min}$ can be derived in a similar way. If Phases 3 and 4 are existing, $Q_{\min} = 0$ and thus the minimum delay is $D_{\min} = \tau$. Otherwise, $Q_{\min}$ is attained within Phase 1. Let $t_{\min}$ be the time instant, where the ACR reaches LCR in Phase 1. We get

$$
\Delta t_{\min} = ACR_1^{-1}(LCR) = \frac{LCR - ACR(t_0)}{LCR \cdot AIR} \quad , \tag{48}
$$

and thus, the minimum queue length $Q_{\min}$ is given by

$$
\begin{aligned}
Q_{\min} &= Q(t_{\min} + \tau_1) \\
&= Q(t_0 + \tau_1) - LCR \cdot \Delta t_{\min} + ACR(t_0) \cdot \Delta t_{\min} + \frac{1}{2}\, LCR \cdot AIR \cdot \Delta t_{\min}^2 \quad .
\end{aligned}
\tag{49}
$$

We can easily derive a lower bound which is independent of the cycle investigated. First, $ACR(t_0)$ is bounded by

$$
ACR(t_0) \geq MCR \quad , \tag{50}
$$

and the queue length at $t_0 + \tau_1$ is lower bounded by

$$
Q(t_0 + \tau_1) \geq Q_L + \int\limits_{x=\Delta t_2 - \tau_f}^{\Delta t_2 + \tau_1} (MCR - LCR)\, dx = Q_L + \tau\,(MCR - LCR) \quad . \tag{51}
$$

By substituting $t_{\min}$, $ACR(t_0)$ and $Q(t_0 + \tau_1)$ in equation (49), we get

$$
\begin{aligned}
Q_{\min} \geq \; & Q_L + \tau\,(MCR - LCR) - \frac{(LCR - MCR)^2}{LCR \cdot AIR} + \\
& \frac{1}{2}\,\frac{(LCR - MCR)^2}{LCR \cdot AIR} \quad ,
\end{aligned}
\tag{52}
$$

which leads to a minimum transmission delay of

$$D_{\min} = \tau + Q_{\min}/LCR \quad . \tag{53}$$

From equation (52) we can conclude that the minimum queue length is strongly dependent on the relation between MCR and LCR and decreases with the RTT τ.

3.2 Homogeneous Sources

The results from the last subsection can easily be generalized for a traffic scenario with homogeneous sources which behave in exactly the same way. I.e. all of the N_{vc} connections which share a common transmission link have the same values of MCR, PCR, ICR, AIR and RDF. They are therefore in phase during the whole life-time of the connections.

To obtain results for such a scenario, the functions for the evolution of the ACRs must be updated. This is due to the change of the arrival rate for backward RM cells, which is now $LCR/(N_{rm} \cdot N_{vc})$, if the switch is fully utilized. In particular, we obtain

$$ACR_1(\Delta t) = ACR(t_0) + \frac{LCR \cdot AIR}{N_{vc}} \Delta t \quad , \tag{54}$$

$$ACR_2(\Delta t) = ACR(t_1) \cdot e^{\frac{LCR}{RDF \cdot N_{vc}} \Delta t} \quad , \tag{55}$$

$$ACR_3(\Delta t) = ACR(t_2) + \frac{LCR \cdot AIR}{N_{vc}} \Delta t \quad , \tag{56}$$

$$ACR_4(\Delta t) = ACR(t_3) e^{\beta \Delta t} \quad . \tag{57}$$

Furthermore, the functions for the evolution of the queue length $Q_i(\Delta t)$ must be substituted by

$$Q_i(\Delta t) = Q(t_{i-1} + \tau_1) + \int_{x=\tau_1}^{\Delta t} (N_{vc} \cdot ACR_i(x - \tau_1) - LCR)\, dx \tag{58}$$

for all phases $i = 1, \ldots, 4$, since we consider N_{vc} traffic sources. Using these modified functions, the evolution of $ACR(t)$, which is identical for all N_{vc} connections, and $Q(t)$ can be computed as described in Section 3.1.

The values for the maximum and minimum queue length are obtained in the same way as outlined in Subsection 3.1.2. For the bounds, we get for example

$$\begin{aligned} Q_{\max} \quad \leq \quad & Q_H + \tau (N_{vc} \cdot PCR - LCR) - \\ & N_{vc} \cdot RDF \cdot \ln \left(\frac{LCR}{N_{vc} \cdot PCR} \right) + N_{vc} \cdot RDF \cdot \left(\frac{N_{vc} \cdot PCR}{LCR} + 1 \right) \quad , \end{aligned} \tag{59}$$

$$Q_{\min} \geq Q_L + \tau\,(N_{vc} \cdot MCR - LCR) - \frac{(N_{vc} \cdot MCR - LCR) \cdot N_{vc} \cdot (LCR - MCR)}{LCR \cdot AIR} + \frac{1}{2}\,\frac{(N_{vc} \cdot (LCR - MCR))^2}{LCR \cdot AIR} \quad . \tag{60}$$

As can be noticed, the number of ABR connections in progress has a serious impact on the buffer capacity requirements.

4 Numerical Examples

With the first example, we demonstrate the accuracy of our analysis, which assumes a continuous increase and decrease of the ACR function and is therefore independent of N_{rm}. In reality, however, the ACR function of each ABR source is stepwise linear. To obtain a scenario where all 4 phases are occurring, we used a single source with $PCR = 15$ *cells/ms*, $MCR = 5$ *cells/ms*, $LCR = 10$ *cells/ms* as well as $AIR = 0.1$ and $RDF = 256.0$. The thresholds of the queue are set to $Q_L = 10$ and $Q_H = 20$ and the RTT is chosen as $\tau = 4$ *ms* where the EFCI switch is located in the center. For the simulation, the modulus for generating RM cells is set to $N_{rm} = 32$. In Figure 4, the ACR and the queue length are depicted for a time interval of 160 *ms*, after the system is in steady state. The analytical results are drawn with solid lines, whereas the simulation results are drawn with dotted lines. We can observe an excellent agreement between the analytical and the simulation results.

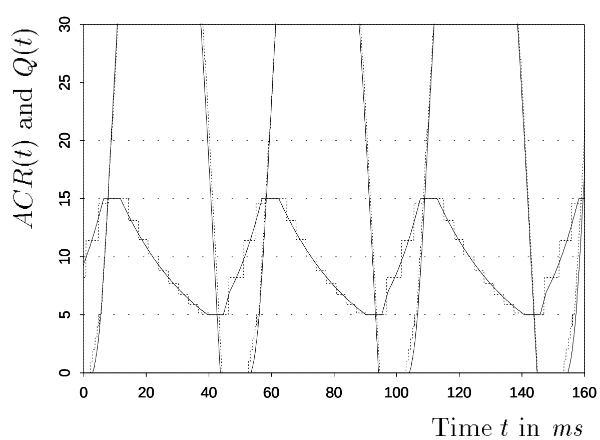

Figure 4: *Accuracy of analytical results*

Next, we show the influence of the PCR on the behavior of the ACR and the queue length in the Figures 5 and 6. We used a single source scenario with $MCR = 5$ *Mbps*, $LCR = 10$ *Mbps*, $AIR = 0.1$, $RDF = 512.0$ and a PCR variing from 12 to 15 *Mbps*. The distance between SES and DES is 100 *km* and the switch is again located in the center. For the queue thresholds we have chosen $Q_L = 10$ and $Q_H = 20$.

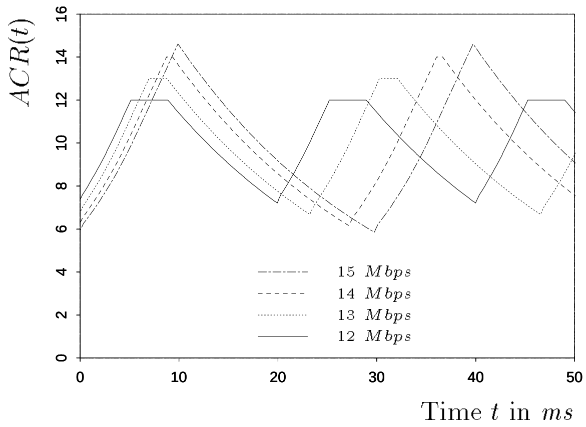

Figure 5: *Influence of PCR on* $ACR(t)$

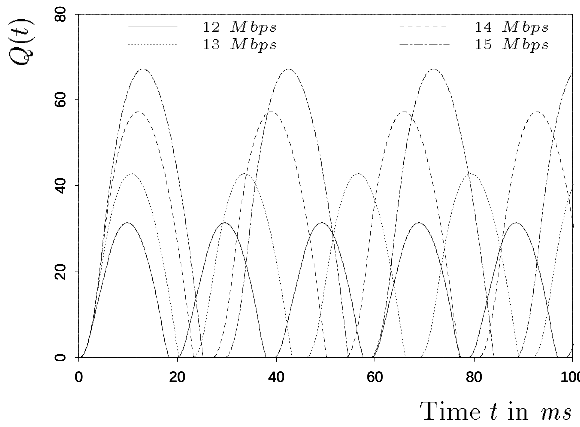

Figure 6: *Influence of PCR on* $Q(t)$

In Figure 5 we observe an increasing cycle length for increasing values of PCR. This is due to the higher values of the maximum queue length (cf. Figure 6), which lead to a longer phase of rate decrease.

If the MCR is variied instead of the PCR, the cycle length remains almost constant. However, by increasing the MCR, the buffer requirements can be reduced. For the same parameters as above, plots for $MCR = 7$, 8 and 9 *Mbps* and $PCR = 15$ *Mbps* are depicted in Figures 7 and 8.

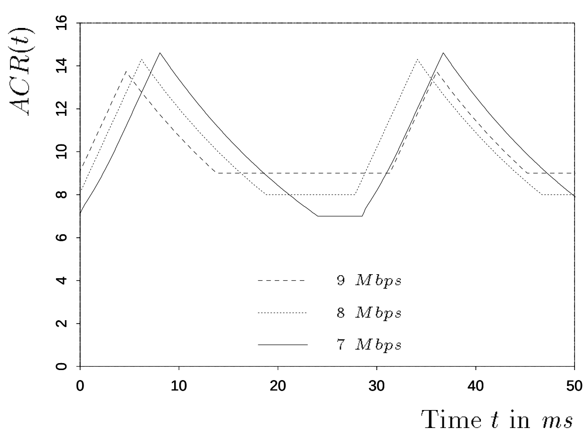

Figure 7: *Influence of MCR on* $ACR(t)$

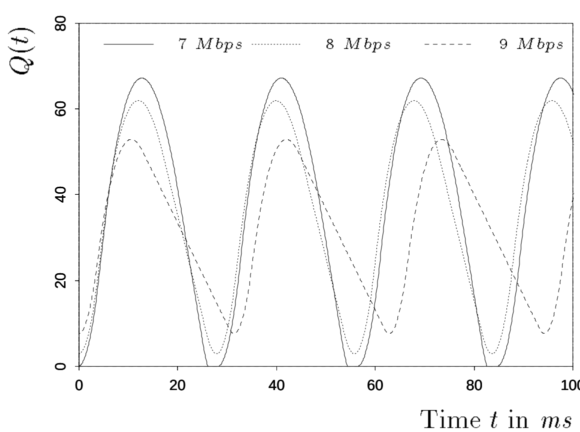

Figure 8: *Influence of MCR on* $Q(t)$

The next two figures show the influence of the number of connections in a homogeneous case. Plots for $N_{vc} = 1$, 5, 10 and 15 ABR sources are shown, where $PCR = 150$ *Mbps*, $MCR = 0$ *Mbps*, $LCR = 100$ *Mbps*, $AIR = 0.1$ and $RDF = 512.0$. The thresholds are $Q_L = 200$ and $Q_H = 300$ and we have an end-to-end delay corresponding to a distance of 100 *km* between SES and DES.

Figure 9 shows a ACR which is getting smoother for increasing numbers of connections, which is a positive effect from traffic management point of view. This is due to the decreasing mean rate, which leads to a longer time interval between to arriving backward RM cells. The consequence is that the rate cannot be decreased or increased that fast during the feedback time. However, the required buffer size increases considerably with the number of connections, which is in contrast a negative effect (cf. Figure 10). This behavior has also been noticed in [6].

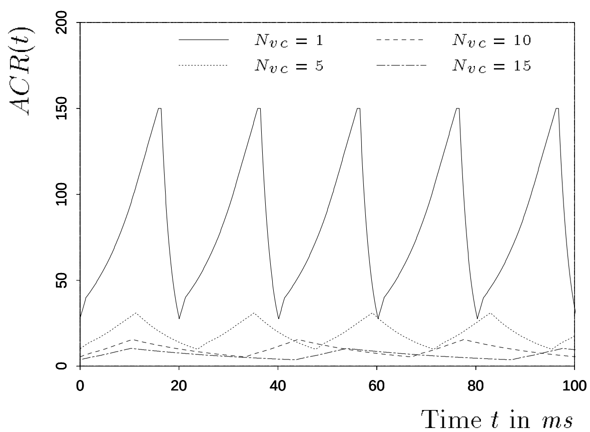

Figure 9: *Influence of* N_{vc} *on* $ACR(t)$

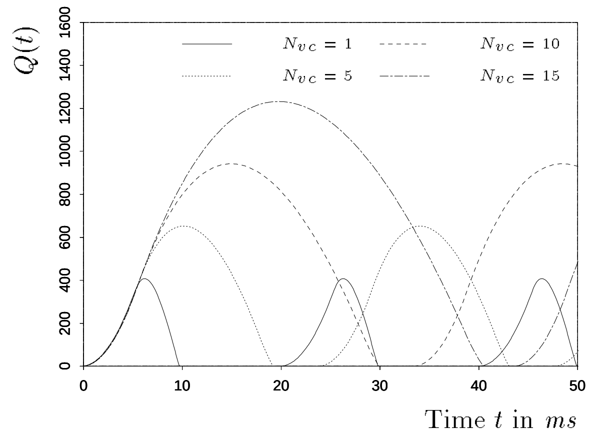

Figure 10: *Influence of* N_{vc} *on* $Q(t)$

With the last two figures, the influence of the RTT is investigated. Figure 11 shows the evolution of the ACR and Figure 12 the evolution of the queue length for a scenario with one source and the same parameter set as above, except the MCR, which is set to $MCR = 50$ *Mbps*. The distance between SES and DES is chosen as 1, 100 and 1000 *km*, where the switch is always located in the center. As expected, the cycle length and the buffer requirements increase with the feedback time.

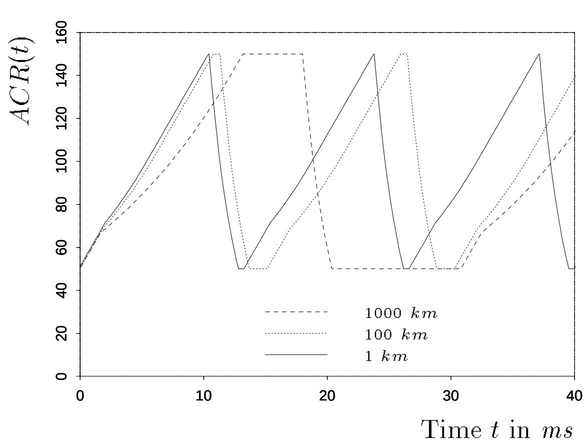

Figure 11: *Influence of RTT on* $ACR(t)$

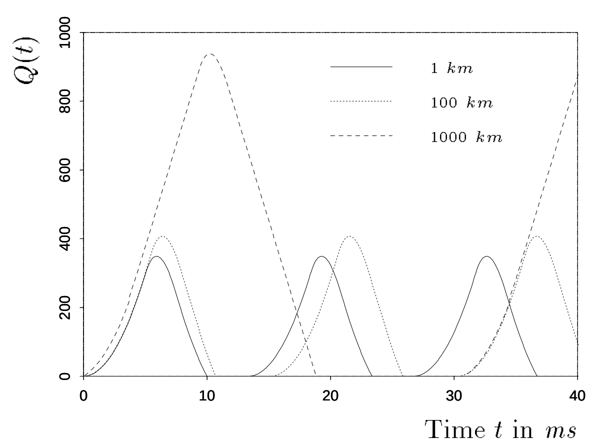

Figure 11: *Influence of RTT on* $Q(t)$

5 Outlook

For data communications in ATM network environments, a closed-loop congestion control mechanism has been suggested by the ATM Forum, which dynamically adjusts the allowed cell rate of each ABR traffic source to the current network load conditions. This is done by resource management cells, which are emitted periodically by the source end system and are returned by the destination end system. Network elements like switches adapt the content of those cells to signal their current congestion situation.

In this paper, we modeled the dynamic behavior of the allowed cell rate and the buffer content in the switches of the feedback-oriented system using a differential equation approach. This leads to results very close to that obtained by simulation. We focused on EFCI switches and considered single source scenarios, as well as mixes of homogeneous sources.

Numerical examples have shown, that the variation of the allowed cell rate and the buffer content as well as the required buffer size depends strongly on almost each of the control parameters of the feedback mechanism. The distance between the end systems, which influences the feedback delay, plays also a dominant role.

To obtain a reasonably good performance of the mechanism for acceptable buffer requirements, the control parameters must be dimensioned appropriatly. Our analysis provides a powerful tool for such a dimensioning, if desired performance objectives are given.

In further research studies, we will investigate traffic mixes with heterogeneous ABR sources, to deal with more realistic scenarios. A companion paper, which deals with explicit rate switches is also in preparation. This will allow a comparison of both switch architectures.

Acknowledgment

The author would like to thank Jörg Domaschke for the stimulating discussions and programming efforts during the course of this work. We also appreciate the support of the Research Institute of the Deutsche Telekom AG (FZ Darmstadt).

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Verantwortlich: Die Vorstände des Institutes für Informatik.

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