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Abstract: The fast computation of blocking probabilities and the resulting capacity is one of the crucial tasks in the planning process of UMTS networks. The admission control in WCDMA networks bases on the momentary interference which includes both own-cell and other-cell interference. Since both interference terms are stochastic values we speak of *soft blocking*. The number of users in the system is not sufficient for deciding whether to accept a new call or not. Instead, it is blocked with a certain probability depending on the number of users in the system, the activity of the users, and the other cell interference. In this paper we present a time efficient algorithm to compute blocking probabilities in a WCDMA network operating with several services. Assuming Bernoulli activity and modelling the other cell interference as a lognormal random variable the blocking probabilities are computed using an approximation based on the Kaufman-Roberts recursion.

1 Introduction

The *Universal Mobile Telecommunication System* (UMTS) is the proposal for third generation wireless networks in Europe. Contrary to conventional second generation systems like GSM which focus primarily on voice and short message services, UMTS provides a vast range of data services with bit rates up to 2Mbps and varying quality of service requirements. This is achieved by operating with *Wideband Code Division Multiple Access* (WCDMA) over the air interface.

The forthcoming introduction of UMTS in Europe requires new paradigms in wireless network planning. In GSM the capacity of a base station (BS) is determined by the number of available frequencies only and hence is independent of the network load. For a number of frequencies the Erlang capacity follows directly from the Erlang-B formula since the GSM network provides mainly voice telephony. In contrast, the capacity of a BS or NodeB in a WCDMA network is interference limited. On the uplink the multiple access interference (MAI) at a BS is caused by all mobile stations (MS) whether they belong to this BS or not. On the downlink the capacity is limited by the transmit power of the BS or by the interference it causes, respectively. The power control mechanisms in both link directions provide that the signals are transmitted with such powers that for each service they are received with nearly equal strength. A detailed examination of the interference on the uplink is no straightforward task. Due to the universal frequency reuse in UMTS, all users both in the considered cell and in the neighboring cells contribute to the total interference, thus influencing the link quality in terms of received bit-energy-to-noise ratio (E_b/N_0).

The planning of WCDMA networks consists of two aspects: The coverage planning and the capacity planning. In contrast to GSM, coverage and capacity cannot be considered as independent terms. In WCDMA a tradeoff between the coverage area and the capacity of a BS exists, see e.g. [1, 2]. The more users are active at a BS, the larger is the MAI at the BS, and the higher are the transmit powers required by the MSs to fulfill their E_b/N_0 requirements. Additionally, due to the restriction of the mobile's transmit power, the coverage area shrinks with an increasing number of users. Attaining a certain coverage area for a BS demands a limitation of the MAI which is done by admission control. The MAI level used as threshold for the acceptance of new calls determines not only the coverage area but also the capacity of the BS. Capacity here means the maximum possible offered load for a BS with a particular service mix while meeting predetermined blocking probabilities. In the iterative planning process for large networks the computation of blocking probabilities at a single BS is only one of many tasks which have to be performed frequently. This demands a time-efficient method capable of providing blocking probabilities with sufficient accuracy. In this paper we propose an approximation fulfilling these requirements.

Since the first introduction of CDMA systems with IS-95 the computation of blocking probabilities and system capacity is an area of intense research. The paper by Gilhousen et al [3] was the first considering the capacity of CDMA systems. The authors computed the probability to maintain the E_b/N_0 requirements for a constant number of users. Viterbi and Viterbi [4] extended the work by modelling the system as an $M/M/\infty$ queue such that the number of users in the system is a Poisson random variable. The activity of a single user is described by a Bernoulli random variable, other-cell interference is included as a multiple of the own-cell interference. In [5], interference is modelled as the sum of a Poisson distributed number of independent random variables. Outage probabilities are determined from the interference. The previously mentioned work considered only a single service. In [6], several services are considered. The CDMA network is described by a product form traffic model whereby a state corresponds to the number of users per service and cell. The admissible states are computed using analysis of statistical multiplexing for each state and the sum of the state probabilities at the border of the admissible region yields the blocking probabilities. In [7], a new kind of approach is used. Again, a single cell is modelled as a queue with a Poisson arrival process and exponential holding times. A state is described by the number of users in the system. In a certain system state a new call is accepted with a probability that depends on the other-cell interference and the actual user activities. However, the analysis is restricted to the case with a single service. In this paper we combine the work of [6] and [7]. From [6] we use the analysis with product form solutions for a system with multiple services. From [7] we take the state-dependent blocking probabilities. However, the two approaches do not match up with each other since the product form solution is not valid if state-dependent blocking probabilities occur. Additionally, the effort for computing state probabilities grows exponentially with the number of services if the product form solution is used. Consequently, the product form solution becomes intractable for practical networks. Instead, we use the approach independently proposed by Kaufman [8] and Roberts [9] to reduce the multi-dimensional state space to a single dimension. We modify the algorithm such that we are able to include state-dependent blocking probabilities and obtain a good approximation of the total blocking probabilities.

The paper is organized as follows. Section 2 describes the system model with the general problem formulation. Section 2.1 explains admission control in WCDMA systems, the assumed interference model, and the load-dependent blocking probabilities. Section 2.2 provides an approach for the exact computation of the total blocking probabilities and Section 2.3 describes the approximation based on a modified Kaufman-Roberts recursion. In Section 3, simulations are used to validate the approximation and, furthermore, the influence of other-cell interference and activity factors are investigated. The paper is concluded in Section 4 with a summary and some proposals for possible extensions and applications of this approximation.

2 Model

The objective of this work is to derive a method for determining the uplink blocking probabilities and the resulting capacity of a WCDMA cell with multiple service classes. The cell is modelled as a loss system with a T-dimensional Markov chain with T being the number of provided service classes. We assume an independent Poisson arrival process for each service class t with an arrival rate λ_t and an exponential holding time with mean $1/\mu_t$. This results in an *offered load* $a_t = \lambda_t/\mu_t$. The user activity at an arrival instant is modelled by a Bernoulli random variable with an activity factor ν_t which is assumed independent at consecutive arrival instants. In the following sections we explain admission control in WCDMA and how blocking probabilities are determined according to the momentary cell load.

2.1 Admission Control in WCDMA Systems

The key feature of WCDMA systems is that all users transmit in the same frequency band and their signals are separated by using orthogonal or pseudo-orthogonal codes. Except for the ideal case when real orthogonal codes are used and no multi-path propagation occurs, a user sees the other users' signal as interference. The interference grows with the number of users in the system and limits the uplink capacity. WCDMA admission control, see e.g. [10], is performed on the basis of the measured *noise rise*. The noise rise is the ratio of the total interference $\hat{I}_0$ to the interference of an unloaded system which corresponds to the thermal noise $\hat{N}_0$. The total interference density comprises the own-cell interference $\hat{I}_{own}$, the other-cell interference $\hat{I}_{other}$, and also the thermal noise. The noise rise is then defined as

$$\text{Noise rise} = \frac{\hat{I}_0}{\hat{N}_0} = \frac{\hat{I}_{own} + \hat{I}_{other} + \hat{N}_0}{\hat{N}_0}, \tag{1}$$

where we use the notation that $\hat{a}$ is a linear value in mW while the corresponding value a is in dBm. The admission control estimates the increase of the noise rise that would be caused by accepting a new connection and blocks it if the result exceeds a predetermined threshold. While the noise rise is a value which is actually measured by a BS, it is not well suited in order to understand the actual system load. A transformation of Eqn. (1) yields the definition of the cell load η:

$$\text{Noise rise} = \frac{1}{\frac{\hat{N}_0}{\hat{I}_{own}+\hat{I}_{other}+\hat{N}_0}} = \frac{1}{1 - \frac{\hat{I}_{own}+\hat{I}_{other}}{\hat{I}_{own}+\hat{I}_{other}+\hat{N}_0}} = \frac{1}{1-\eta}. \tag{2}$$

A cell load equal to 1 defines the pole capacity of a WCDMA cell. We define η_{max} as the cell load corresponding to the noise rise value used for admission control. On the arrival of a new call of service t the admission control algorithm estimates the additional load α_t. This load is based on the negotiated bearer properties, i.e. bit rate and maximum error rates. WCDMA admission control consequently accepts an incoming connection if the estimated cell load η_{est} is below the cell load threshold η_{max}:

$$\eta_{est} = \eta + \alpha_t < \eta_{max} \Leftrightarrow \frac{\hat{I}_{own} + \hat{I}_{other}}{\hat{I}_{own} + \hat{I}_{other} + \hat{N}_0} < \eta_{max} \tag{3}$$

We can see that the acceptance of a new call depends on both the own-cell interference and the other-cell interference. In the next section we show how to derive the interferences from the power control equation.

2.1.1 Interference model

The total interference received at a BS determines the uplink capacity of the WCDMA cell. The thermal noise is a fixed value and in a first step we assume the other-cell interference as known. The own-cell interference density corresponds to the sum of the received powers of the $\bar{m} = (m_1, \ldots, m_T)$ active MSs power controlled by the considered BS divided by the system bandwidth W:

$$\hat{I}_{own} = \sum_{t=1}^{T} \frac{m_t \hat{S}_t}{W} \tag{4}$$

Assuming perfect power control, the received MS powers $\hat{S}_t$ have to fulfill the power control equation

$$\hat{\varepsilon}_t^* = \frac{\frac{\hat{S}_t}{R_t}}{\hat{I}_{own} + \hat{I}_{other} + \hat{N}_0 - \frac{\hat{S}_t}{W}} \tag{5}$$

with $\hat{\varepsilon}_t^*$ and R_t being the target-E_b/N_0 and the bit rate of the MSs operating with service t. With perfect power control all MSs of one service require an equal received power. Solving Eqn. (5) for this power $\hat{S}_t$ yields

$$\hat{S}_t = W \frac{\hat{\varepsilon}_t^* R_t}{W + \hat{\varepsilon}_t^* R_t} \left(\hat{N}_0 + \hat{I}_{own} + \hat{I}_{other} \right). \tag{6}$$

By replacing $\hat{S}_t$ in Eqn. (4) we obtain the own-cell interference density as

$$\hat{I}_{own} = (\hat{I}_{own} + \hat{I}_{other} + \hat{N}_0) \sum_{k=1}^{t} m_t \omega_t \tag{7}$$

with ω_t defined as

$$\omega_t = \frac{\hat{\varepsilon}_t^* R_t}{W + \hat{\varepsilon}_t^* R_t}. \tag{8}$$

Note that the variable ω_t contains all service-specific parameters and is sufficient to describe the influence of the service type on the interference. Solving Eqn. (7) for $\hat{I}_{own}$ leads to

$$\hat{I}_{own} = \frac{A(\bar{m})}{1 - A(\bar{m})} (\hat{I}_{other} + \hat{N}_0), \tag{9}$$

where $A(\bar{m})$ is defined as

$$A(\bar{m}) = \sum_{t=1}^{T} m_t \omega_t. \tag{10}$$

Eqn. (9) defines the own-cell interference depending on the variable $A(\bar{m})$ which is the sum of the ω_t of the users active in the cell. Using this definition we are able to write the cell load η in the following way:

$$\eta = \frac{\frac{A(\bar{m})}{1-A(\bar{m})}(\hat{I}_{other} + \hat{N}_0) + \hat{I}_{other}}{\frac{A(\bar{m})}{1-A(\bar{m})}(\hat{I}_{other} + \hat{N}_0) + \hat{I}_{other} + \hat{N}_0} = \frac{\hat{N}_0}{\hat{N}_0 + \hat{I}_{other}} A(\bar{m}) + \frac{\hat{I}_{other}}{\hat{N}_0 + \hat{I}_{other}} \tag{11}$$

Defining $\bar{m}^{t+}$ as $(m_1, \ldots, m_t + 1, \ldots, m_T)$, the admission control condition (3) is reformulated to

$$\frac{\hat{N}_0}{\hat{N}_0 + \hat{I}_{other}} A(\bar{m}^{t+}) + \frac{\hat{I}_{other}}{\hat{N}_0 + \hat{I}_{other}} < \eta_{max} \Leftrightarrow \quad A(\bar{m}^{t+}) + (1 - \eta_{max}) \frac{\hat{I}_{other}}{\hat{N}_0} < \eta_{max}. \tag{12}$$

This equation has the interesting property that own-cell load and other-cell load are separated. The own-cell load is equal to $A(\bar{m})$ and in consistence with [11] the pole capacity is reached for $A(\bar{m}) = 1$ if the other-cell interference is neglected. Without other-cell interference $A(\bar{m}^{t+})$ is equivalent to η_{est}. Consequently, we define the own-cell load for $\bar{m}$ users as

$$\eta_{own}(\bar{m}) = A(\bar{m}) \tag{13}$$

and the load per service as $\alpha_t = \omega_t$. Since an equal own-cell load may occur for different user numbers $\bar{m}$ we write η_{own} without indicating the user number if we use the term own-cell load in a general context. In the next section we derive the blocking probability for a new call arriving in a system state with own-cell load η_{own}.

2.1.2 Local Blocking Probabilities

Blocking occurs if the total interference exceeds a defined maximum level or if the cell load exceeds the maximum threshold η_{max}, respectively. In this case WCDMA admission control rejects the request for a new connection. The probability $\beta_t(\eta_{own})$ for this event, i.e. the blocking of a new call of service t when arriving at an instance with own-cell load η_{own}, is called *local blocking probability* in the remainder. Local blocking probabilities are computed by

$$\beta_t(\eta_{own}) = P\left(\eta_{own} + \omega_t + (1-\eta_{max})\frac{\hat{I}_{other}}{\hat{N}_0} \geq \eta_{max}\right). \tag{14}$$

In this equation all variables except the other-cell interference are known. We model the other-cell interference as a random variable which is independent at consecutive arrival events. In general, the distribution of the momentary other-cell interference may depend on the momentary own-cell load. In the following, however, we assume the other-cell interference as a lognormal random variable with parameters μ and σ that do not depend on the own-cell load. This assumption is justified by [12, 13] which propose an iterative approach used to calculate the first and second moments of the other-cell interferences in a UMTS networks with an arbitrary BS layout and a non-homogeneous spatial traffic distribution. The resulting other-cell interferences are shown to be lognormal distributed. In [13] coverage areas are determined under the assumption that the other-cell interference is independent of the current own-cell load. The results match well with simulated results.

With this premises blocking for a given own-cell load occurs if the other-cell interference exceeds a certain limit. Let η_{other} be a random variable for the other-cell load defined by:

$$\eta_{other} = (1-\eta_{max})\frac{\hat{I}_{other}}{\hat{N}_0}. \tag{15}$$

The other-cell load is also a lognormal random variable with distribution function $\Gamma(x)$ and parameters:

$$\mu_\Gamma = \mu + \log(1-\eta_{max}) - \log(\hat{N}_0) \quad \text{and} \quad \sigma_\Gamma = \sigma, \tag{16}$$

So, we obtain for the local blocking probabilities

$$\beta_t(\eta_{own}) = \begin{cases} 1 - \Gamma(\eta_{max} - (\eta_{own} + \omega_t))), & \text{if } \eta_{max} - (\eta_{own} + \omega_t) > 0 \\ 1, & \text{else} \end{cases} \tag{17}$$

According to this equation we are able to compute blocking probabilities depending on the own-cell load. Fig. 1 shows the local blocking probabilities depending on the estimated cell load. The curves are plotted for different mean other-cell interferences with a constant variation coefficient of 1. A very high other-cell interference causes blocking probabilities of almost 20% even for an own-cell load of zero, i.e. without any user in the system. The curve for $E[\hat{I}_{other}] = 10^{-50}$mW represents the case without other-cell interference. All other curves with more realistic values for $E[\hat{I}_{other}]$ between 5×10^{-19}mW and 6×10^{-18}mW are between these two extremes. Note that in the legend of figures we use I instead of $\hat{I}_{other}$.

Let us summarize the status of our analysis. Local blocking probabilities depending on the own-cell load are determined according to Eqn. (17). In the following we aim at computing the steady state distribution

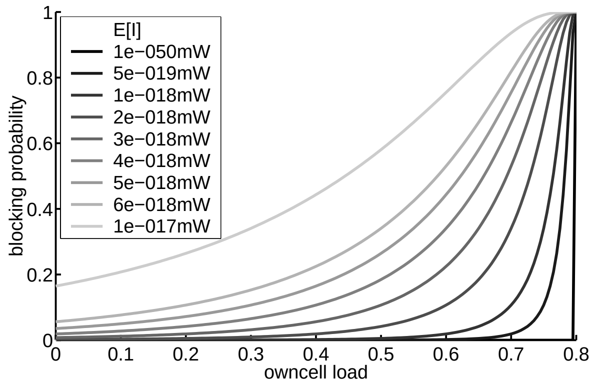

Figure 1: Local blocking probabilities

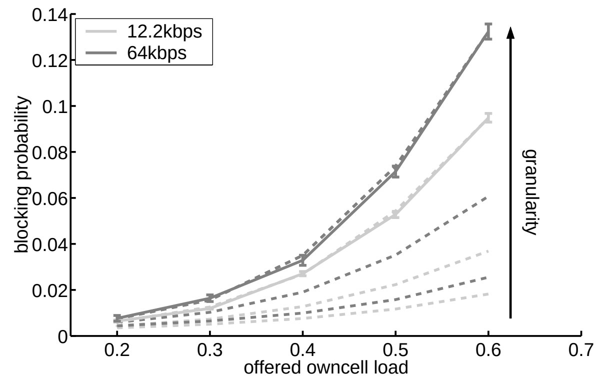

Figure 2: Approximation accuracy for different granularities

of the own-cell load in the system to obtain the total blocking probabilities using the theorem of total probability. In order to achieve this we propose a model for the exact computation of the steady state distribution and subsequently introduce an approximation based on the Kaufman-Roberts recursion.

2.2 Exact Computation of Total Blocking Probabilities

In general our model is based on a T-dimensional Markov chain which may be computed using the product form solution provided that no local blocking occurs. A state is defined by the number $\bar{n}$ of users in the system and the state space Ω is restricted by a maximum number of users large enough to avoid hard blocking. We define a transition rate matrix for the exact computation of blocking probabilities since the application of the product form solution is not valid with local blocking. The introduction of local blocking probabilities for each transition $\bar{n} \to \bar{n}^{t+}$ leads to reduced transition rates. The Markov chain is adapted in order to get an appropriate model behavior, similar as in [7]. Figure (3) shows the state diagram of a modified one-dimensional Markov chain with local blocking if a single service is considered.

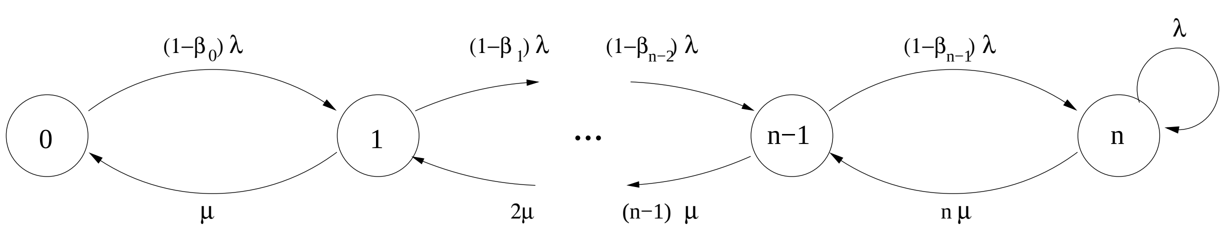

Figure 3: State diagram of a modified one-dimensional Markov chain

The calculation of blocking probabilities in the current state requires the knowledge of the distribution of the momentary user activity. The variable n_t denotes the number of users with service t in the systems and m_t are the active connections. The local blocking probability $B_t(\bar{n})$ for a new call of service t in state

$(\bar{n})$ is

$$B_t(\bar{n}) = \sum_{\bar{m} \leq \bar{n}} P(\bar{m}|\bar{n}) \beta_t(\eta_{own}(\bar{m})). \tag{18}$$

According to Eqn. (13), $\eta_{own}(\bar{m})$ is the own-cell load and the blocking probability $\beta_t(\eta_{own}(\bar{m}))$ results from Eqn. (17). The number of active users with service t is binomially distributed:

$$P(m_t|n_t) = \binom{n_t}{m_t} (1-\nu_t)^{n_t - m_t} \, \nu_t^{m_t}. \tag{19}$$

A new call of service t arriving in state $(\bar{n})$ is accepted with probability $1 - B_t(\bar{n})$. Accordingly the transition rate from $(\bar{n})$ to $(\bar{n}^{t+})$ decreases from λ_t to $(1 - B_t(\bar{n}))\lambda_t$. We define the transition rate matrix Q as

$$Q(f(\bar{n}), f(\bar{n}^{t\pm})) = \begin{cases} (1 - B_t(\bar{n}))\lambda_t & \text{for } \bar{n} \to \bar{n}^{t+} \\ n_t \mu_t & \text{for } \bar{n} \to \bar{n}^{t-} \end{cases} \tag{20}$$

and $Q = 0$ else. The injective index function f maps the state space Ω to $\mathbb{N}$. The steady state distribution vector $\bar{X}$ is computed by solving the equation system:

$$Q\bar{X}^T = 0 \quad \text{and} \quad \sum_{\bar{n} \in \Omega} \bar{X}(f(\bar{n})) = 1 \tag{21}$$

From the steady state distribution the total blocking probability $P_{block}(t)$ for service t results as

$$P_{block}(t) = \sum_{\bar{n} \in \Omega} \bar{X}(f(\bar{n})) B_t(\bar{n}) \tag{22}$$

This method is numerically intractable and not suitable for multi-dimensional scenarios with large state spaces, since the size of Q grows exponentially with the number of services.

2.3 Approximation of the Total Blocking Probabilities

The Kaufman-Roberts recursion is applicable for systems with a shared resource and a set of services with different resource requirements. The algorithm exploits the fact that the blocking of an arriving call does not explicitly depend on the number of users in the system but on the resources they occupy. Consequently, it is sufficient to determine the steady state distribution of the resource occupancy. The Kaufman-Roberts algorithm reduces the T-dimensional state space to a one-dimensional state space by combining the states which occupy the same amount of resources to one state. For a system with capacity C and T services with resource requirements r_t and an offered load a_t the steady state probability $p(j)$ that j resources are occupied is given as

$$\tilde{p}(j) = \begin{cases} 0 & , \text{ for } \ 0 < j \ \text{ or } \ j > C \\ 1 & , \text{ for } \ j = 0 \\ \sum_{t=1}^{T} a_t \frac{r_t}{j} \tilde{p}(j - r_t) & , \text{ else} \end{cases} \qquad \text{and} \qquad p(j) = \frac{\tilde{p}(j)}{\sum_{c=0}^{C} \tilde{p}(c)} \tag{23}$$

The service specific blocking probabilities are then

$$P_t^{block} = \sum_{j=0}^{r_t-1} p(C-j) \tag{24}$$

In WCDMA networks we can interpret the cell load η as shared resource and the load per service ω_t as resource requirement. The application of the Kaufman-Roberts algorithm postulates a discrete shared resource and discrete service requirements. Additionally, the algorithm is beneficial only if a single state (j) combines many multi-dimensional states $(\bar{n})$. Thus, we discretisize the cell load by introducing a cell load unit g of which η_{max} should be an integer multiple. The resulting capacity and resource requirements are then:

$$C = \frac{\eta_{max}}{g} \text{ and } r_t = \text{round}\left(\frac{\omega_t}{g}\right) \tag{25}$$

The cell load unit g controls the granularity of the Kaufman-Roberts state space. The smaller g is, the larger is the state space and the better is the approximation. In the simple case of no other-cell interference and no user activity, the Kaufman-Roberts recursion yields exact steady state probabilities except for the error caused by the cell load discretization. Fig. 2 shows total blocking probabilities for such a scenario considering two services with 12.2 kbps and 64 kbps bit rates and corresponding target-E_b/N_0 values of 5dB and 4dB, respectively. The solid curves depict the simulation results and the dashed curves the analytic results for different granularities. The finer the granularities the better the results match. The two smallest cell load units of 0.005 and 0.001 lead to almost identical results and curves. Therefore, we choose $g = 0.005$ as cell load unit for the numerical results presented in Section 3.

The integration of the local blocking probabilities into the Kaufman-Roberts recursion is not directly possible. A state corresponds to the resources occupied when all users are active. The state description does not provide the number of active users and the maximum occupied resource is not sufficient to determine the distribution of the resource occupancy. We introduce the random variable Λ for the number of occupied resources. Still assuming that no local blocking occurs the probability $\Lambda(c|j)$ that c resources are occupied in state j is computed recursively:

$$\Lambda(c|j) = \sum_{t=1}^{T} P_t(j)\,[\nu_t \Lambda(c-r_t|j-r_t) + (1-\nu_t)\Lambda(c|j-r_t)]. \tag{26}$$

The probability $P_t(j)$ denotes the probability that state j is reached by a new call of service t and is given by:

$$P_t(j) = \frac{\tilde{p}(j-r_t)a_t\frac{r_t}{j}}{\tilde{p}(j)}. \tag{27}$$

Figure 4 illustrates the algorithm for the recursive calculation of the resource occupancy distribution Λ. The example relates to a system of two services with resource requirements 1 and 2. On the left, the Kaufman-Roberts state space is shown. The transitions between neighbored states are for the service with requirement 1 and the transitions overleaping one state are for the service with requirement 2. For reasons of clarity the labelling of the transition arrows is omitted. The right side shows the recursive calculation of the occupancy distribution. In state (0) without users obviously no resources are occupied, i.e. $\Lambda(0|0) = 1$. In state (1) we have one user of service 1 which is either active or not. That leads to $\Lambda(0, 1) = 1 - \nu_1$ and $\Lambda(1, 1) = \nu_2$. For state (2) it becomes more complicated as we have either two users with service one or one user with service two. The probability for the first case is $P_1(2)$ and the probability for the second case is $P_2(2)$. Let us pick out the probability that no user is active in state two. With probability $P_1(2)$ we have the occupancy distribution of state (1) plus a new user with service one that is either active or not. And with probability $P_2(2)$ we have the occupancy distribution of state (0) plus a new user with service two that is either active or not. Putting that together we obtain

$$\Lambda(0|2) = P_1(2)\Lambda(0|1)(1 - \nu_1) + P_2(2)\lambda(0|0)(1 - \nu_2). \tag{28}$$

In the scheme this calculation is indicated by the arrows. A solid arrow means the probability that the new call is active and the dashed arrow that the new call is passive. Let us consider another special case, the probability $\Lambda(2, 4)$ that 2 resources are occupied in state (4). This case is quite general since it composes four ways to obtain this resource occupancy. The new user may either be of service one or two and it may be either active or passive. The four arrows running into the circle labelled with $\Lambda(2|4)$ indicate the recursive computation of these four possibilities.

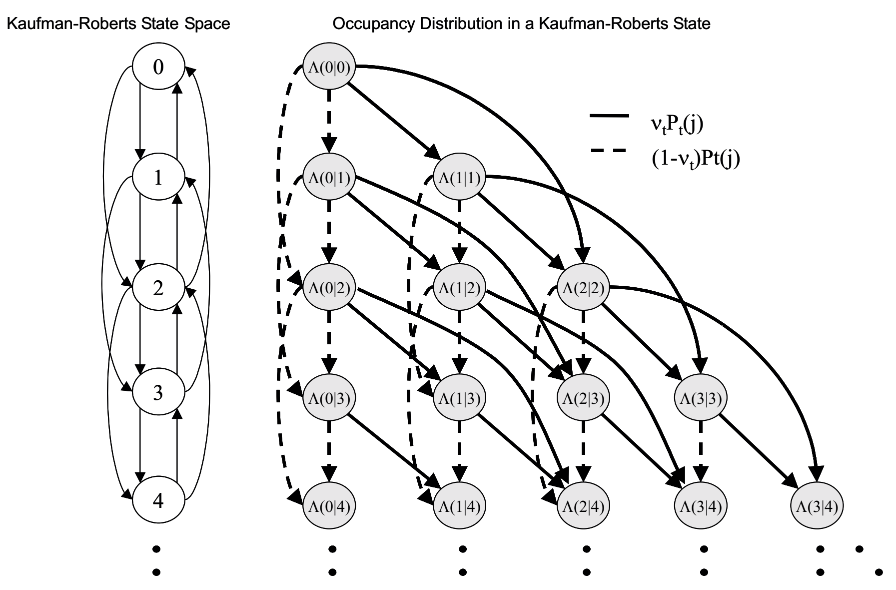

Figure 4: Recursive computation of the resource occupancy

This recursive computation of the resource occupancy distribution is exact if the product form solution is applicable, i.e. if no local blocking occurs. With local blocking probabilities included the transition rate is reduced and we have to modify the recursion.

Let us start with state (0) and an empty system. In this state holds:

$$\tilde{p}(0) = 1, \Lambda(0|0) = 1, \Lambda(i|0) = 0 \text{ for } i \neq 0, \text{ and } B_t(0) = \beta_t(0) \tag{29}$$

We initialize $\tilde{p}(j) = 0$ for $j < 0$. The unnormalized probabilities and local blocking probabilities for state j are computed recursively. The unnormalized state probability $\tilde{p}(j)$ follows from Eqn. (23) extended by local blocking:

$$\tilde{p}_(j) = \sum_{t=1}^{T} \tilde{p}(j - r_t)(1 - B_t(j - r_t))a_t \frac{r_t}{j} \tag{30}$$

This probability depends on $\tilde{p}(j - r_t)$ and $B_t(j - r_t)$ which are known for all states c with $c < j$. In order to determine the local blocking probabilities $B_t(j)$ the probability that state j is reached from state $j - r_t$ is required:

$$P_t(j) = \frac{\tilde{p}(j - r_t)(1 - B_t(j - r_t))a_t \frac{r_t}{j}}{\tilde{p}(j)} \tag{31}$$

The resource occupancy distribution is computed according to Eqn. (26) and the local blocking probabilities $B_t(j)$ are derived using the theorem of total probability:

$$B_t(j) = \sum_{c=0}^{j} \Lambda(c|j)\beta_t(c). \tag{32}$$

The iterative computation of unnormalized state probabilities and local blocking probabilities is done until either the local blocking probabilities approach one or further states are unreachable due to hardware restrictions. After determining all unnormalized state probabilities $\tilde{p}(j)$ the steady state probabilities $p(j)$ follow by normalization:

$$p(j) = \frac{\tilde{p}(j)}{\sum_{c=0}^{C} \tilde{p}(j)} \tag{33}$$

The total blocking probability for a service t is the sum of all state probabilities $p(j)$ multiplied with the local blocking probabilities $B_t(j)$ for all reachable states:

$$P_{block}(t) = \sum_{j=0}^{j_{max}} B_t(j)p(j) \tag{34}$$

3 Numerical Results

All numerical results are calculated for a WCDMA system with a chip rate of 3.84 Mcps and a thermal noise $N_0 = -174dBm$. We consider bearer services with 12.2kbps, 64 kbps, and 144kbps and target-E_b/N_0 of 5dB, 4dB, and 3dB, respectively. The maximum load η_{max} is set to 0.8.

3.1 Validation by Simulation

This section is dedicated to the validation of our results. In Section 2.2 we presented an exact analysis, however, it is not applicable in realistic scenarios. Instead, we use a event-discrete simulation for the validation. In the simulation we use the same assumptions as in the approximation, i.e. the user activities and the other-cell interferences are independently determined at every arrival event. We want to point out that the intention of the simulation is not the verification of our assumptions but to demonstrate the accuracy of the approximation.

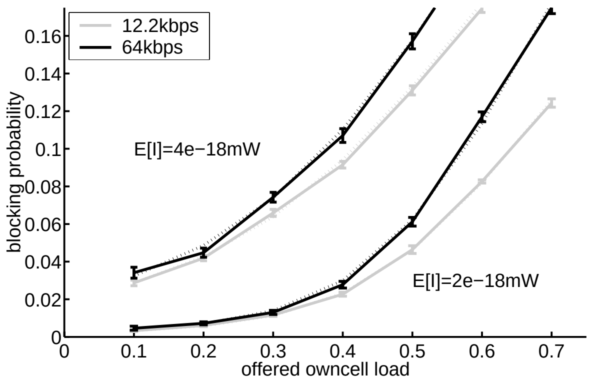

Figure 5: Different other-cell interference levels

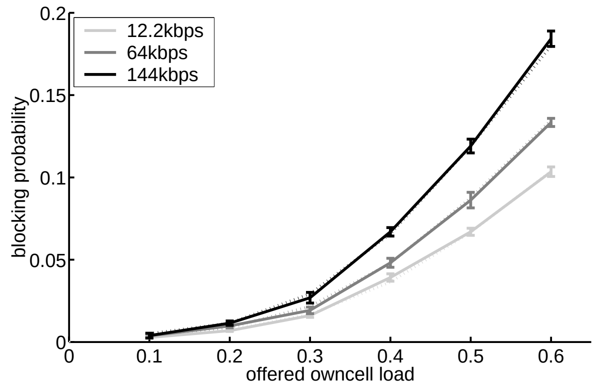

Figure 6: Scenario with three service classes

Fig. 5 shows blocking probabilities for a scenario with two service classes of 12.2kbps and 64kbps, respectively. The traffic mix is set to 75% 12.2kbps 25% 64kbps. The connections are assumed to be Always-ON, i.e. $\nu_t = 1$. The solid lines represent the simulation results with 90% confidence intervals and the dotted lines depict the approximated values. On the x-axis you find the offered own-cell load which is defined as

$$\text{offered own-cell load} \quad = \sum_{t=1}^{T} \nu_t a_t \omega_t. \tag{35}$$

The influence of the other-cell interference is evident due to the fact that the curves for a mean other-cell interference of 4×10^{-18}mW are noticeably higher than for their counterparts with 2×10^{-18}mW. The figure also illustrates the accuracy of the modified Kaufman-Roberts algorithm even for a high other-cell interference.

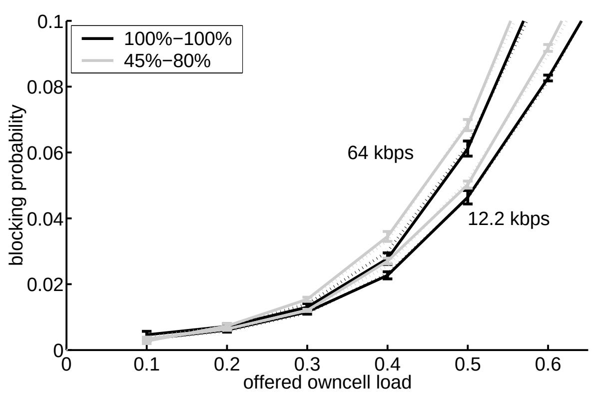

Figure 7: Influence of activity on blocking probabilities

A scenario with three service classes is shown in Fig. 6. The service mix is 50% 12.2kbps, 25%

64kbps, and 25% 144kbps. The mean other-cell interference is set to $E[\hat{I}_{other}] = 2 \times 10^{-18}$mW with a variation coefficient of 1. The activity factors for the three service classes are set to 0.45, 0.3 and 0.8, respectively. The high requirements of the high-speed service class lead to increased blocking probabilities in comparison to the other classes. With growing offered own-cell load the gap between the 144kbps class and the 64kpbs increases faster than the gap between the 64kbps and the 12.2kbps service class. This indicates that the high-speed services are more sensitive for higher cell loads than lower bit rate services. The simulation and the analysis match well. This is remarkable since in this scenario we have used parameters which are critical for the approximation: A high and considerably varying other-cell interference, multiple services with different values for ω_t, and low activities. The approximation is exact for either no other-cell interference and Always-ON users or for a single service. The parameters chosen for Fig. 6 are therefore a challenging scenario for our approximation.

Fig. 7 illustrates the impact of Bernoulli activity on blocking probabilities. Both scenarios are calculated with equal loads but different activity factors per service. $E[\hat{I}_{other}]$ is set to 2×10^{-18}mW with variation coefficient 1. The scenario with 100% activity for both service classes yields lower blocking probabilities than its counterpart with a 45% to 80% mix. This indicates that it is not sufficient to simply reduce the Poisson arrival rate by the activity factors, since this approach corresponds to the Always-ON scenario.

3.2 Parameter Studies

Let us examine some interesting parameters and their influences on the system. Fig. 8 shows the plot of two scenarios with two service classes, the first with activity factors 0.45 and 0.8 and the second with both service classes Always-ON. All blocking probabilities shown in this figure are computed with an offered own-cell load of 0.4 and an other-cell interference with $E[\hat{I}_{other}] = 2 \times 10^{-18}$mW and a variation coefficient of 1. The varying parameter is the traffic mix between the two service classes, it ranges from solely 12.2kbps to solely 64kbps. On the x-axis we have the share of the 12.2kbps service. Two effects can be observed: First, the blocking probabilities decrease with an decreasing contingent of the higher bit rate service. This holds true for both scenarios with and without activity. Second, as already observed in Fig. 7, Bernoulli activity has a noticeable influence on the blocking probability if comparing scenarios with equal loads.

In Fig. 9 the influence of the variation coefficient of the other-cell interference with otherwise equal scenario parameters is illustrated. The total offered load, i.e. offered own-cell load plus mean other-cell load $E[\eta_{other}]$ is kept constantly at 0.6. On the x-axis the share of the offered own-cell load increase until the other-cell load vanishes. The lowest curve considers deterministic other-cell load. The blocking probabilities increase slightly with the own-cell load. With increasing variation coefficients for$\hat{I}_{other}$ the results show a behavior reciprocal to the first curve: The blocking probabilities decrease with an increasing

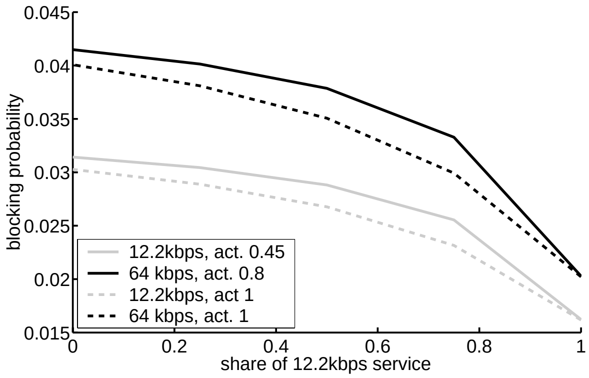

Figure 8: Influence of the traffic mix on blocking probabilities

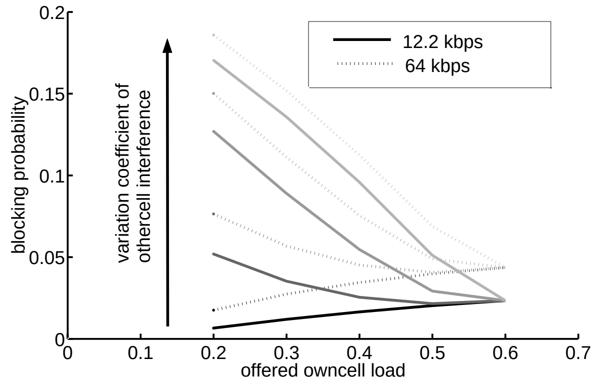

Figure 9: Influence of the variation coefficient of the other-cell interference

contingent of own-cell load. The reason is that the part of the total load with higher variance also has a higher influence on the blocking probabilities. It can be concluded that not only the quantity of the other-cell interference but also the quality expressed by the standard deviation is an important factor in network planning.

4 Conclusion

The admission control in UMTS networks accepts or blocks calls depending on the current interference level which includes both own-cell and other-cell interference. We show that the blocking condition can be expressed in terms of own-cell load and other-cell load. Assuming the other-cell interference as lognormal random variable independent of the own-cell load allows us to calculate local blocking probabilities. The term local blocking denotes the event that a new call is blocked in the instance of a certain own-cell load. Local blocking probabilities are typical for WCDMA networks with soft blocking, i.e. without a hard limit for the number of users in the system or the number of occupied resources. The shared resource which corresponds to the cell load is stochastic. These local blocking probabilities prevent the application of the product form solution so exactly determining the blocking probabilities requires the inversion of the transition rate matrix which is numerically intractable.

As a solution we develop an algorithm based on the Kaufman-Roberts recursion which allows a time-efficient approximation of the blocking probabilities. We validate our results by simulation and show that the approximation yields accurate results even for large other-cell interferences and low user activities. These are the especially interesting cases since without other cell activity and Always-ON users our analysis yields exact results. There are some promising extensions of our model for future research. It would be interesting to combine the computation of blocking probabilities with the iterative calculation of the first and second moment of the other-cell interference according to [12]. Adapting the computation of the local blocking probabilities to the downlink would allow a similar analysis for the downlink cell capacity.

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