Dynamic VeloSLAM – Preliminary Report on 3D Mapping of Dynamic Environments

Ming Li, Wei Li, Jian Wang, Qingquan Li, and Andreas Nüchter

***Abstract*—3D mapping using point cloud registration is a basic inevitable problem for many applications, especially for modeling of large scale complicated environments. This paper presents a novel approach for mapping highly dynamic environments, i.e., we present a system capable for mapping road traffic scenarios. Given 3D laser scans acquired at a high frame rate and no other sensor input, a 3D map is built by removing dynamic parts of the scene and estimating the ego-motion of the vehicle precisely at the same time. We extend the well-known ICP algorithm for HDL-64 laser scan data and build a system for solving the simultaneous localization and mapping problem in urban road scenarios. This paper presents initial results on two data sets.**

I. Introduction

THIS paper focuses on the problem of solving the simultaneous localization and mapping (SLAM) problem in urban road scenarios. The proposed algorithms allow to digitize large environments fast and reliably without any intervention. A car equipped with a Velodyne HDL-64 laser scanner acquires 3D data in urban scans – in heavy traffic. We consider that the Velodyne scanner is moving along a path, and that there are many moving vehicles and pedestrians around. Figure 1 shows some camera images of the scenarios we want to map.

The general problem we want to solve is to consistently align overlapping 3D point clouds, captured by a high-speed moving Velodyne HDL-64 Lidar, into a complete model. To create such a model, the scans have to be merged into one coordinate system. This process is called registration. If the car carrying the 3D scanner were precisely localized, the registration could be done directly based on the pose of the car. As the pose of the car is unknown, the geometric structure of overlapping 3D scans has to be considered for registration. However, this structure is changing, due to change of position of the other moving objects. Therefore, moving objects need firstly to be identified and be removed. We use a semantic-driven approach for solving this task of identifying dynamic objects in 3D scans. The overall system is called dynamic VeloSLAM.

Ming Li, Wei Li, and Jian Wang are with the Multimedia Research Center, Computer School, Wuhan University, China. liming751218@whu.edu.cn Qingquan Li is with the Transportation Research Center, Wuhan University, China. Andreas Nüchter is Automation and Machine Vision Group, Jacobs University Bremen, Germany, andreas@nuechti.de

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II. State of the art

A. 3D point cloud registration

The goal of registration is to find the relative position and orientation of one 3D scan, called the scene D, to another, called model scan M. The most famous method for solving this task is the iterative closest point (ICP) algorithm originally developed by Besl and McKay [6], by Chen and Medioni [10] and by Zhang [38] at the same time in 1991. The algorithm relies on minimizing the following cost function:

$$E(\mathbf{R},\mathbf{t}) = \frac{1}{N}\sum_{i=1}^{N} \left\| \mathbf{m}_i - (\mathbf{R}\mathbf{d}_i + \mathbf{t}) \right\|^2. \quad (1)$$

All corresponding points are represented in a tuple $(\mathbf{m}_i, \mathbf{d}_i)$ where $\mathbf{m}_i \in M \subset \hat{M}$ and $\mathbf{d}_i \in D \subset \hat{D}$. Two things have to be calculated: First, the corresponding points, and second, the transformation $(\mathbf{R}, \mathbf{t})$ that minimizes $E(\mathbf{R}, \mathbf{t})$ on the basis of the corresponding points. The ICP algorithm uses closest points as corresponding points. A sufficiently good starting guess enables the ICP algorithm to converge to the correct minimum. Four closed form solutions are known for minimizing the ICP error function [24], such as a SVD-based solution [1], a solution based on orthonormal matrices [19], a unit quaternion [18], and a dual quaternion solution [34]. Rusinkiewicz and Levoy [30] provide a detailed analysis

Figure 1. Typical scenario, where we aim at precise 3D mapping. See also http://youtu.be/bHaZpQ_5wg8

of efficient variants of the ICP approach, discussing point-to-point vs. point-to-plane metrics, nearest neighbor assignment strategies and different rejection rules. In the case that the scene includes points which are not part of the model (from a non-overlapping or previously occluded area), wrong correspondences are assigned for these points which might lead to an erroneous result [13]. The simplest solution is the use of a distance threshold. Corresponding tuples are rejected if their Euclidean distance exceeds this value. Several strategies are possible to determine suitable thresholds, e.g., a gradual decreasing threshold with respect to the iteration step. In general, these thresholds increase the registration performance on partially overlapping point clouds significantly, but are difficult to parameterize for not getting stuck in a local minimum. Many extensions to the ICP approach have been published addressing the determination of valid point correspondences from overlapping parts. Chetverikov et al. proposed the *Trimmed ICP (TrICP)* approach [12]. It employs a parameter representing the degree of overlap, i.e., the number of corresponding points N.

Recently, alternatives to ICP have been presented. These include [28], which uses a squared distance function and the normal distribution transform (NDT) [7], [26]. An extension to the NDT algorithm to globally consistent scan matching is still missing.

While registering several 3D data sets using the ICP algorithm errors sum up. These errors are due to imprecise measurements and small registration errors. Therefore, globally consistent scan matching algorithm aim at reducing these errors.

B. Globally consistent 3D scan matching

Chen and Medioni [11] aimed at globally consistent range image alignment when introducing an incremental matching method, i.e., all new scans are registered against the so-called metascan, which is the union of the previously acquired and registered scans. This method does not spread out the error and is order-dependent. Bergevin et al. [5], Stoddart and Hilton [33], Benjemaa and Schmitt [3], [4], and Pulli [29] present iterative approaches. Based on networks representing overlapping parts of images, they use the ICP algorithm for computing transformations that are applied after all correspondences between all views have been found. However, the focus of research is mainly 3D modeling of small objects using a stationary 3D scanner and a turn table; therefore, the used networks consist mainly of one loop [29], where the loop closing has to be smoothed. On the other hand, approaches like TORO and HOG-Man by Grisetti et al. [14], [16], [15] focus on optimizing large networks of 3D scans and lack a proper data association, i.e., scan matching module. A probabilistic approach to such a module was proposed by Williams et al. [35], where each scan point is assigned a Gaussian distribution in order to model the statistical errors made by laser scanners. This causes high computation time due to the large amount of data in practice. Krishnan et al. [23] presented a global registration algorithm that minimizes the global error function by optimization on the manifold of 3D rotation matrices. Borrmann et al. first presents a globally consistent scan registration framework called 6DSLAM [9] and Nüchter et al. presents a study of parameterizations for the rigid body transformations of general SLAM problem [27].

Figure 2. SmartV-II: An Autonomous car developed at Wuhan University.

C. Mapping dynamic environments

Only a little work has been done in the area of mapping dynamic environments, especially in 3D. Seminal work was done by Dirk Schultz where a mobile robot explored and mapped an indoor environment [31]. The localization was done using Monte Carlo, and dynamic objects were detected using the idea of templates. Promising work was done by Becker et al. in [2]. Using 2D laser scans, the approach successfully managed to detect dynamic objects and mark them as such in the resultant map. However, the vehicle was stationary when the scans were taken. Further approaches were presented in [37], [8]. Bobruk and Austin use grid mapping for localization, and then scan subtraction to find dynamic objects [8]. Yu et al., Himmelsbach et al., and Kondaxakis et al. use scan matching and/or odometry to localize robot, and then scan comparison to detect dynamic objects [37], [17], [21].

The approach described by Wu and Sun [36] uses motion detection to remove dynamic objects from the final map and create accurate 3D maps using SLAM. Currently, the best results were achieved by Katz et al [20]. The approach used segmentation and clustering of data, and scan matching for localization, and gave very good results for a forward looking laser sensor. Full 6D SLAM in dynamic environments is still not solved.

III. Preliminaries

The system used in this paper consists of an automatic vehicle developed at Wuhan University, called SmartV-II (cf. Fig. 2). In this study we use only the Velodyne HDL-64 laser sensor mounted on top of the car at a height of 2.10][m.

Figure 3 presents the algorithmic overview. The most important part is the red box, i.e., the method for removing dynamic objects. However, mapping a static environment does not need to consider this part.

IV. SLAM with a Velodyne scanner in static environments

We use our 6D SLAM method [9]. Its basis is a fast and reliable scan matching algorithm for ICP and Lu/Milios

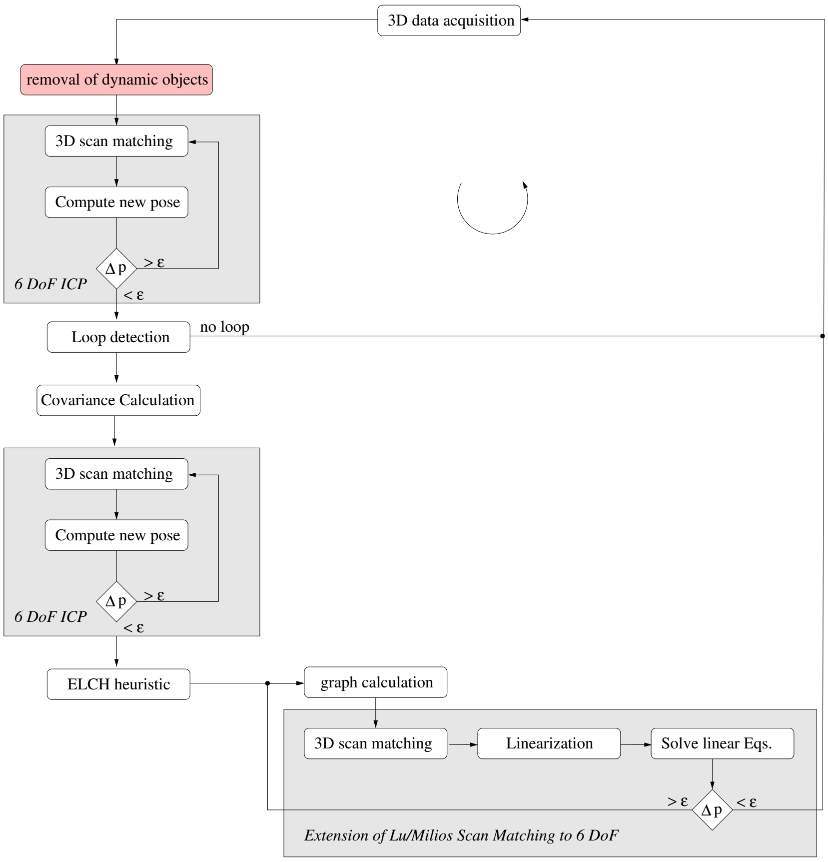

Figure 3. Overview of the VeloSLAM method.

style relaxation [25]. Please see [9] for the estimation of the covariance matrices and the equation solver, the latter one is our current SLAM back end. The back end is based on a sparse Cholesky decomposition.

A. 6D SLAM based ICP Based Scan Matching

We use the ICP algorithm to calculate the transformation while the car is acquiring a sequence of 3D scans by minimizing Eq. (1). Consider a car driving along a path, and traversing $(n+1)$ 3D scan poses $\mathbf{X}_0, \ldots, \mathbf{X}_n$. A straightforward method for aligning several 3D scans taken from the poses $\mathbf{X}_0, \ldots, \mathbf{X}_n$ is *pairwise ICP*, i.e., matching the scan taken from pose $\mathbf{X}_1$ against the scan from pose $\mathbf{X}_0$, matching the scan from $\mathbf{X}_2$ against the one from $\mathbf{X}_1$, and so on.

Once a closed loop is detected, a 6 DoF graph optimization algorithm for global relaxation is employed, a variant of GraphSLAM. Our method relies on a notion of the uncertainty of the poses, calculated by the registration algorithm. We extend the probabilistic approach first proposed in [25] to 6 DoF. For a more detailed description of the extension refer to [9]. However, before we apply the graph optimization, we use the explicit loop closing heuristic (ELCH) [32] at the point, where a loop closing is detected.

ELCH aims at reducing the run time of our mapping system such that it performs fast in large environments. ELCH efficiently closes the loop and aligns scans consistently by avoiding or reducing the iterative 3D scan matching over *all* scans, in the extension of Lu/Milios Scan Matching to 6 DoF, i.e., in principle, it is possible to neglect the bottom gray box in Fig. 3. ELCH operates with 6D poses, i.e., is able to handle robot motion with six effective degrees of freedom (translation and rotation). A detailed description of the heuristics is given in [32].

B. Mapping results in static environments

Data for this experiment was acquired at Wuhan university at night to ensure that there are no moving objects in the scene. The static dataset contains 953 3D point clouds, acquired every 0.1 second, each containing 138240 3D points.

For processing data of a Velodyne scanner, we preprocess the data using octree-based subsampling. Starting from a cube surrounding the scan data, we subdivide the cube recursively until its side length is smaller than 0.1 m and select randomly one point from the cube. Using these down-sampled 3D point clouds we use incremental ICP-based scan matching for obtaining an initial pose estimate. The threshold for the maximal allowed point-to-point distance is set to 0.2 m. Fig. 4 (top left) shows the mapping result using this plain vanilla ICP.

Since the points on the ground do not provide any information for the registration task, ICP is only successful, after removing these points below 1.80 m from the scanner, the registration becomes much more precise. Fig. 4 (bottom) presents a result.

While registering several 3D point clouds small registration inaccuracies sum up and therefore the loop is not closed.

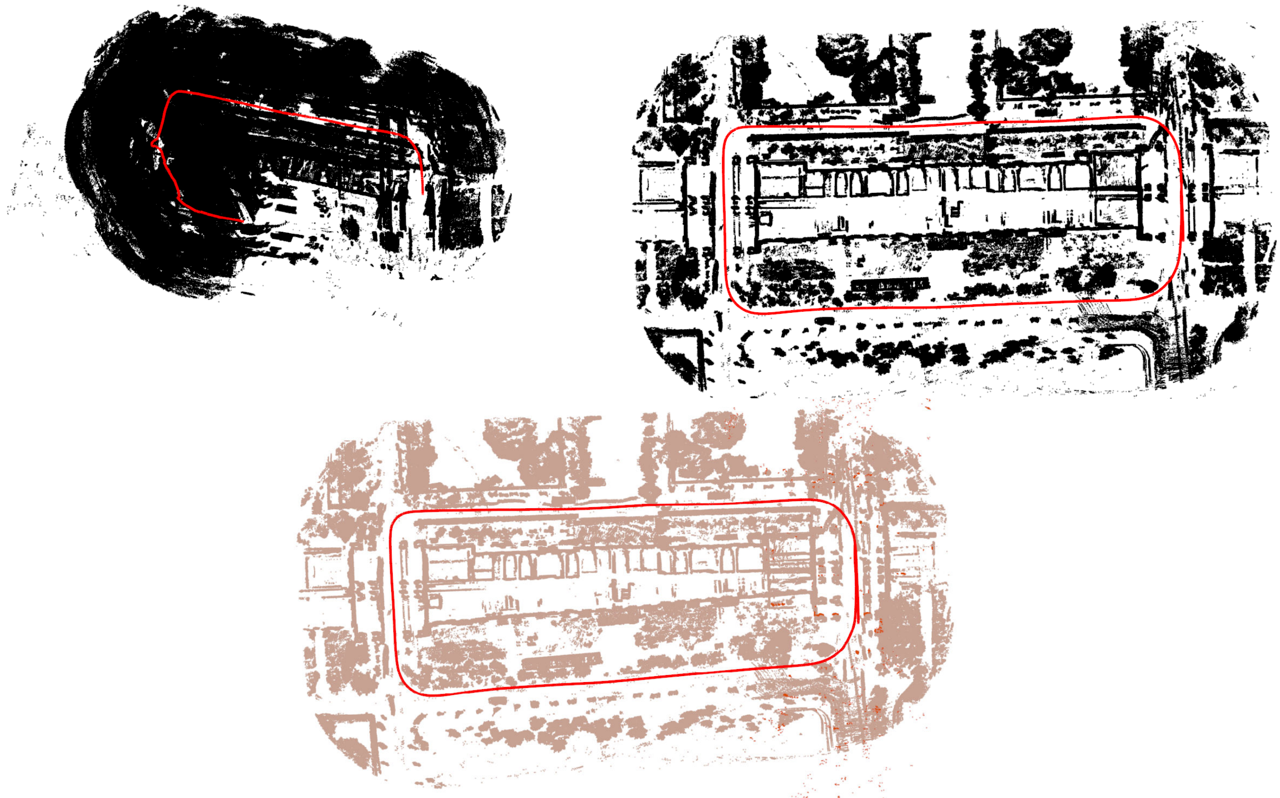

Figure 4. Mapping results using a Velodyne scanner. Top left: Plain vanilla ICP. Top right: Precise registration result using ICP and globally consistent scan matching. Bottom: ICP-based registration after removing ground points.

Table I
RUNTIMES IN SECONDS FOR THE STATIC ENVIRONMENT DATA SET ON AN INTEL(R) CORE(TM)2 QUAD CPU Q9450 @ 2.66GHZ.

algorithm	run time
ICP scan matching	225.28 s
ELCH heuristic	0.41 s
Global relaxation	740.55 s

Since the car is returning to its initial position, we can form a network of overlapping scans, i.e., a pose graph. The loop closing is triggered whenever the pose estimate of the car reaches a position within 7.5 m to a previously encountered position. We optimize the resulting graph using the globally consistent scan matching method [9], again with a maximal allowed point-to-point distance is set to 0.2 m for 50 iterations. The final result is given in Fig. 4 (top right) and the run times are given in Table I.

V. SLAM IN DYNAMIC ENVIRONMENTS

The process of point cloud registration by our dynamic VeloSLAM can be formulated as two steps. First, we remove the points of ground and moving objects. Second, we perform ICP-based 6D SLAM of the points labeled as static.

A. Ground and moving objects detection

To build a unified framework for dynamic VeloSLAM, we do not extend our previously discussed straight-forward ground point cutoff. Our general method for point cloud segmentation is to put all 3D points into a 2D occupancy grid data structure aligned to the (X, Z)-axis of the coordinate system. Then, the features of points in the grid are computed, such as the variance (σ). The variance indicates the tendency of variable dispersion, so if the variance of the cell is below a certain threshold, we treat this cell as flat and then delete all points belonging to that cell. In our approach, the grid cell size is 0.1 m $\times$ 0.1 m and the threshold of variance for ground points is set to 0.23.

When segmenting a 3D point cloud, a problem arises with all objects standing on the ground. For example, the feet of a human have roughly the same height value as the ground at the point he is standing on. The feet and the floor form only a crease edge, no jump edge. This problem is solved by ground point removal described above.

After the ground segmentation, one scan of point cloud was divided into many separated objects. In a typical urban environment, there is a wide variety of objects such as vehicles, pedestrians, buildings, etc. We use low-level geometric features, i.e., PCA features, for classification of the objects. This approach is motivated by the tensor voting approach of Kornprobst and Medioni [22]. The local spatial point distribution over some neighboring area is captured by the decomposition into principal components of the covariance matrix of the 3D point's position. The size of the neighborhood is considered as the support region and defines the scale of the features. The symmetric positive definite covariance matrix for a set of N 3D points $(\mathbf{p}_i) = ((x_i, y_i, z_i))$ with $\bar{p} = \frac{1}{n}\sum_1^n \mathbf{p}_i$ is defined in Equation (2).

$$\frac{1}{N}\sum_{i=1}^{N}(\mathbf{p}_i - \bar{\mathbf{p}})(\mathbf{p}_i - \bar{\mathbf{p}})^T. \qquad (2)$$

The matrix is decomposed into principal components ordered

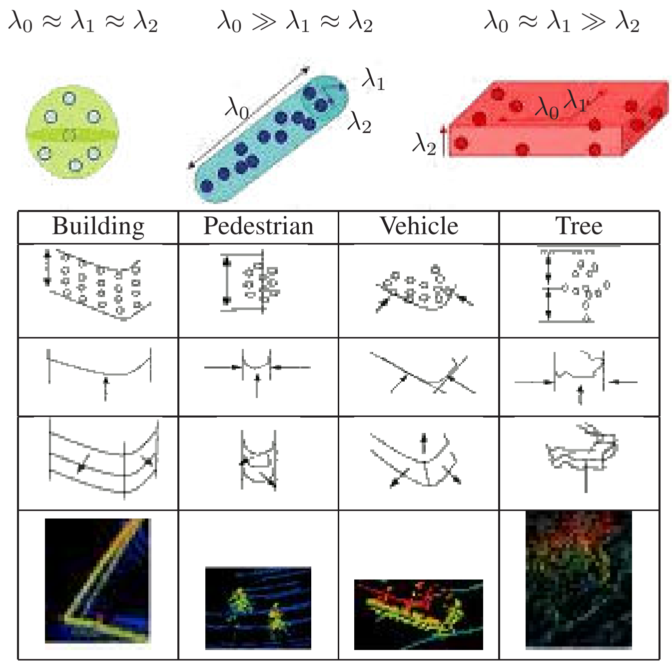

Figure 5. Illustration of the PCA features.

by increasing eigenvalues. $\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2$ are the eigenvectors corresponding to the eigenvalues $\lambda_0, \lambda_1, \lambda_2$ respectively, i.e., $\lambda_0 \geq \lambda_1 \geq \lambda_2$.

In the case of scattered points, we have $\lambda_0 \approx \lambda_1 \approx \lambda_2$ and no dominant direction is found. In the case of a linear structure, the principal direction will be the tangent at the curve, with $\lambda_0, \lambda_1, \lambda_2$. Finally, in the case of a solid surface, the principal directions are aligned with the surface normal with $\lambda_0, \lambda_1, \lambda_2$ and $\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2$ span the local plane of observations. Those features, named scatter-ness, linear-ness and surface-ness, are linear combinations of the eigenvalues. Figure 5 (top) illustrates the three features used to find moving objects such as vehicles and pedestrians.

To be invariant to the rotation of occupancy grid, we identify a reproducible orientation for the interest points. After removing of ground and dynamic objects we proceed with 6D SLAM as described in section IV-A.

B. Results in dynamic environments

To validate our approach, we collected a data set on the road from Wuhan to Huangshi . We select a roundabout in the road as main experimental scene. Figure 6 shows the traffic circle. We processed 2000 3D scans each containing 2160×64 range data points. Figure 7 (top left) shows the result of ICP matching without object and ground removal in comparison to the final results (top right). Interestingly, the plain vanilla ICP is able to recover the basic orientation, while the estimated rotations and translations are too small. This might be due to the fact, that a high overlap and low error value of Eq. (1) is obtained by registering scans exactly at the same pose. Figure 7 (bottom) shows two intermediate mapping results, before and after loop closing. The car continued driving as given in Figure 7 (top right). When comparing the mapping result with a given map, i.e., with Figure 6 one sees, that the result is a circle. However, the traveled distance is still too small and therefore, the resulting circle is too large. Neither with the ICP algorithm nor with loop closing nor with global relaxation is possible to obtain a accurate map. The detailed runtimes of this experiment are given in Table II.

Figure 6. The traffic circle used for mapping evaluation.

Table II
RUNTIMES IN SECONDS FOR THE DYNAMIC ENVIRONMENT DATA SET ON AN INTEL(R) CORE(TM)2 QUAD CPU Q9450 @ 2.66GHZ.

algorithm	run time
removal of objects	2.1 s
ICP scan matching	450.24 s
ELCH heuristic	0.51 s
Global relaxation	186.08 s

VI. CONCLUSIONS

This paper has presented results on 3D mapping with a Velodyne scanner. Furhtermore, we show our initial mapping results in highly dynamic environments. Road traffic as well as stop and go traffic provide a highly challenging scenario and using the Velodyne HDL-64 laser scanner one is in principle able to acquire sufficient information for reliably 3D mapping. The crucial part is the removal of dynamic objects. Needless to say a lot of work remains to be done. In future work, we will concentrate on improving the removal of dynamic objects. To this end, we plan to implement a hierarchal, i.e., octree-based, tracking of objects.

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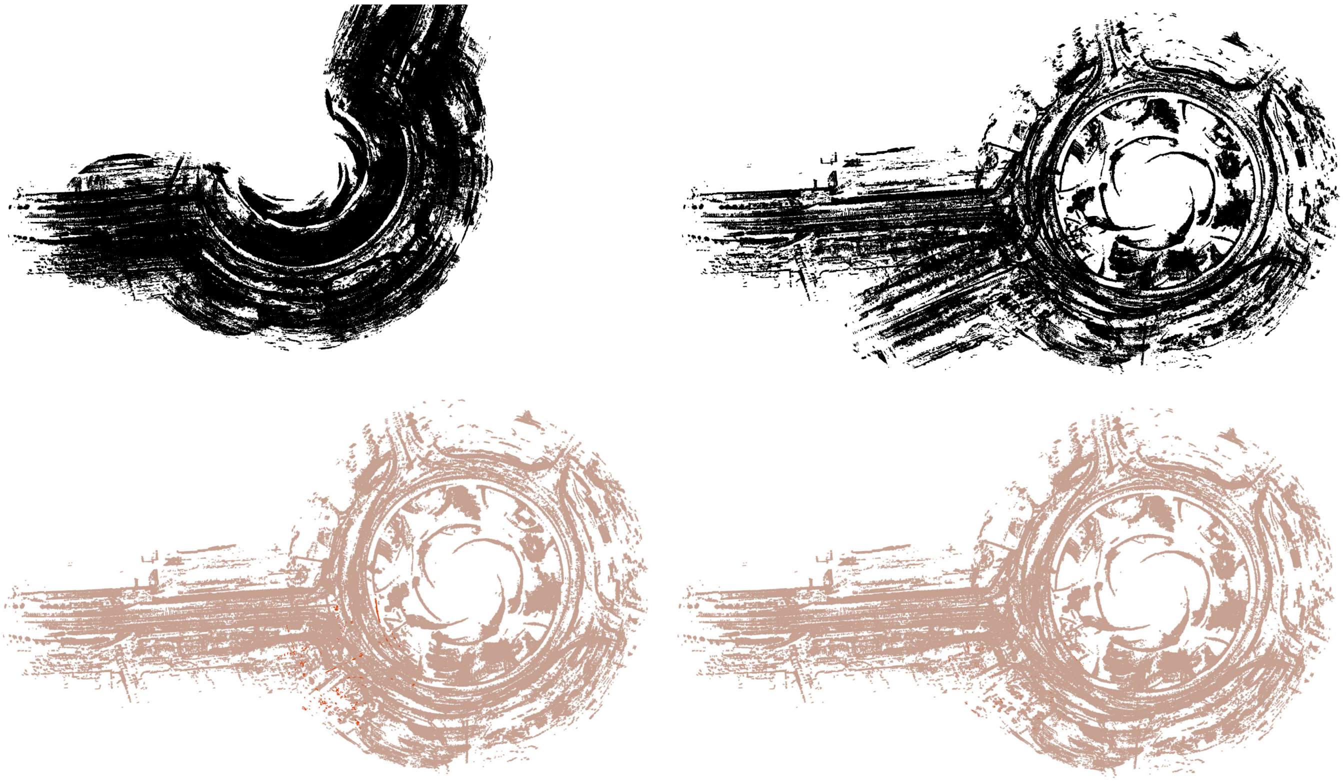

Figure 7. Mapping results using a Velodyne scanner. Top left: Plain vanilla ICP. Top right: Registration result using ICP and globally consistent scan matching. Bottom: ICP-based registration after removing ground points.

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