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Fluctuations of biological rhythms: Self-organized criticality and predictability?

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Abstract

Temporal fluctuations which cannot be explained as consequences of statistically independent random events are found in a variety of physical and biological phenomena. The fluctuations of these systems can be characterized by a power spectrum density $S(f)$ decaying as f^{-b} at low frequencies with an exponent $0.5 \leq b \leq 1.5$. We present a new approach to describe the individual biorhythm of humans using data from a colleague who has kept daily records for four years of his state of well-being applying a fifty-point magnitude category rating scale. This time series $\{R(t_i)\}$ was described as a point process by introducing discriminating rating levels r and s for the occurrence of $R(t_i) \geq r$ ('ups') and $R(t_i) \leq s$ ('downs').

For $b < 1$ a new method to estimate the low frequency part of $S(f)$ was applied using counting statistics without applying Fast Fourier Transform. The method applied reliably discriminates these types of fluctuations from a random point process with $b = 0.0$. It is very tempting to speculate that the neuronal/humoral mechanisms at various levels of the nervous system underlying the perception of different values of the subjective state of well-being are expressions of a self-organized critical state. But the most important result of the present study is the finding of a scaling region $1d \leq \Delta t \leq 15d$ for the 'ups' and 'downs' where $S(f)$ is decaying as f^{-b} with $b \approx 0.7$. Therefore, based on one's own monitored biorhythm for a given time period it should be possible to predict future episodes with a certain probability by applying methods of nonlinear time series analysis or modified feed-forward neural networks learning with the backpropagation algorithm.

1 Introduction

A variety of phenomena in nature exhibit temporal fluctuations in the absence of intentional stimulation which cannot be explained as consequence of statistically independent random events. It has been shown that temporal fluctuations found in phenomena as different as membrane currents, earthquakes, sunspot activity, light emitted from quasars, sand falling through an hour glass, traffic flow, heart beat or breathing activity can be characterized by their power spectrum density $S(f)$ decaying as f^{-b} at low frequencies with $0.5 \leq b \leq 1.5$. This behavior of the temporal fluctuations of a system described by its $S(f)$ is called $1/f$-noise.

Recently, Bak, Tang and Wiesenfeld [4] suggested that the large fluctuations in time characterized as $1/f$-fluctuations and the self-similarity in space might both be manifestations of a self-organized critical state. Self-organized criticality (SOC) describes the tendency of some open dissipative many-body systems to drive themselves spontaneously to a critical state with no characteristic time or length scales without any fine-tuning by external fields: hence the criticality is self-organized. This is in contrast to the criticality of equilibrium systems undergoing phase transition only at a critical external field, such as temperature, pressure, electrical or magnetic field. The idea provides a unifying concept for large scale behavior in systems with many degrees of freedom operating persistently far from equilibrium at or near a threshold of instability, so to speak at the 'border to chaos' [2].

The SOC phenomenon is expected to be quite uni-

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versal and we assume that it is the underlying principle of some biological many-body systems. We present a new approach to describe the individual biorhythm of humans using data from a colleague who has kept standardized daily records for four years of his state of general well-being applying a fifty-point magnitude category scale and analyze the temporal fluctuations by estimating the power spectrum density in its low frequency range to characterize the self-similar temporal rating sequences.

2 Methods

The subjective intensity of well-being was measured with a linear category scaling procedure (category partitioning, [10]) with five categories each subdivided in ten steps: 1 - 10: very strong 'down', 11 - 20: strong 'down', 21 - 30: moderate 'down'/moderate 'up', 31 - 40: strong 'up', 41 - 50: very strong 'up'. Thus, the subject could, after choosing a major category, fine-tune the rating of well-being by choosing a number within that main category. In general, the daily ratings were performed at 6.00 a.m. and stored for subsequent analysis. Occasionally fluctuations within a day of the subjective well-being were observed, but were not monitored and therefore neglected in this analysis. The time series of the daily ratings $R(t_i)$ (Fig.1) can be described as a point process by introducing discriminating rating levels for the occurrence of $R(t_i) \geq r$, e.g. for the occurrence of 'ups' (cf. Fig. 2) and $R(t_i) \leq s$, e.g. for the occurrence of 'downs' (cf. Fig. 3).

Usually $S(f)$ is obtained by Fast Fourier Transform (FFT). To avoid the well-known problems in using FFT for the obtained point process, we used a new simple method based on counting statistics [20] to analyze the low frequency part of $S(f)$ of the recorded ratings of human general well-being.

The series of recorded ratings after introducing a discriminating rating level is considered to be a point process described as

$$y(t) = \sum_{i=1}^{n} \delta(t - t_i) \qquad (1)$$

in which $\delta(t - t_i)$ represents Dirac's delta function, and t_i is the time of occurrence of a particular $R(t_i) \geq r$ or $R(t_i) \leq s$ within the train of n events. In the absence of severe intentional stimulation $y(t)$ is assumed to be statistically stationary. Another statistical variable derived from Eq. (1) is the actual num-

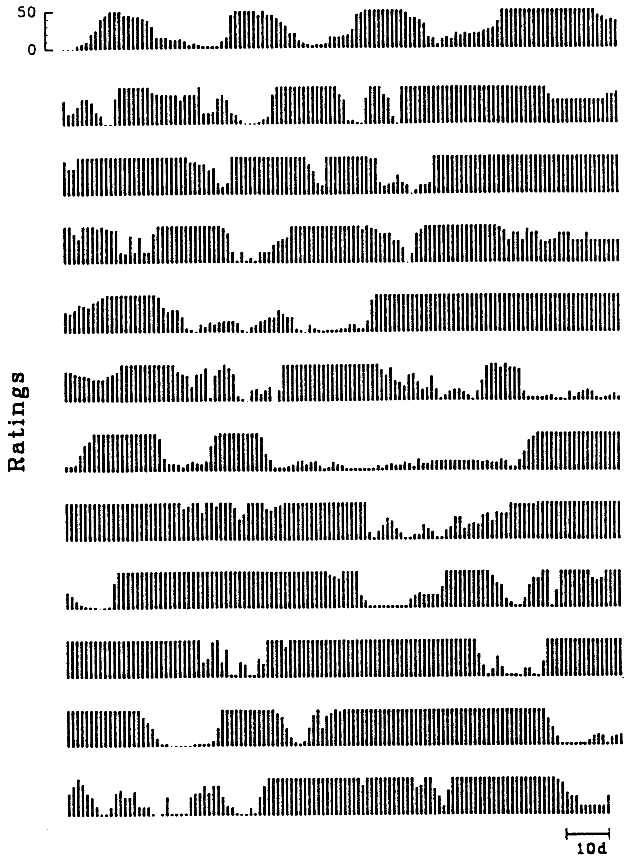

Figure 1: Biological rhythm. Daily ratings of the subjective well-being for four years using a linear category scaling procedure.

ber of events $N(\Delta t)$ occurring in a time interval Δt ranging from t_1 to t_2. Thus, $N(\Delta t)$ can be expressed as

$$N(\Delta t) = \int_{t_1}^{t_2} \sum_{j}^{\nu} \delta(t - t_j)\, dt. \qquad (2)$$

The variance of counts $Var[N(\Delta t)]$ is the so-called variance-time curve. Its second time derivative is related to the auto-covariance function of y, $C_y(\Delta t)$ by

$$C_y(\Delta t) = \frac{1}{2}(Var[N(\Delta t)])'' \qquad (3)$$

[8] and therefore the key to determine the low frequency part of the spectrum $S_y(f)$ is to experimentally obtain $Var[N(\Delta t)]$ [20]. If the variance-time curve follows within certain limits a power law

$$Var[N(\Delta t)] \propto (\Delta t)^{1+b} \text{ with } b < 1, \qquad (4)$$

then it can be shown using the Wiener-Chinchin theorem that the spectrum $S_y(f)$ scales as

$$S_y(f) \propto f^{-b} \qquad (5)$$

within $f_{min} < f < f_{max}$ [13].

The variance-time curve is defined by the variance of counts for time intervals of length Δt as

$$Var[N(\Delta t)] = \langle N^2(\Delta t)\rangle - \langle N(\Delta t)\rangle^2 \qquad (6)$$

with $\langle ... \rangle$ denoting expectation values. For estimating $Var[N(\Delta t)]$, the entire observation time T is divided into k counting windows of duration Δt with $T = k\Delta t$ and the variance of counts is determined for this particular window Δt. This is repeated for different values of Δt. The results were plotted as $Var[N(\Delta t)]$ versus Δt on a log-log scale and fitted by linear regression using the least square method.

3 Results

In Fig. 1 the whole data set is shown, i.e., the daily ratings $R(t_i)$ for four years. It is obvious from the

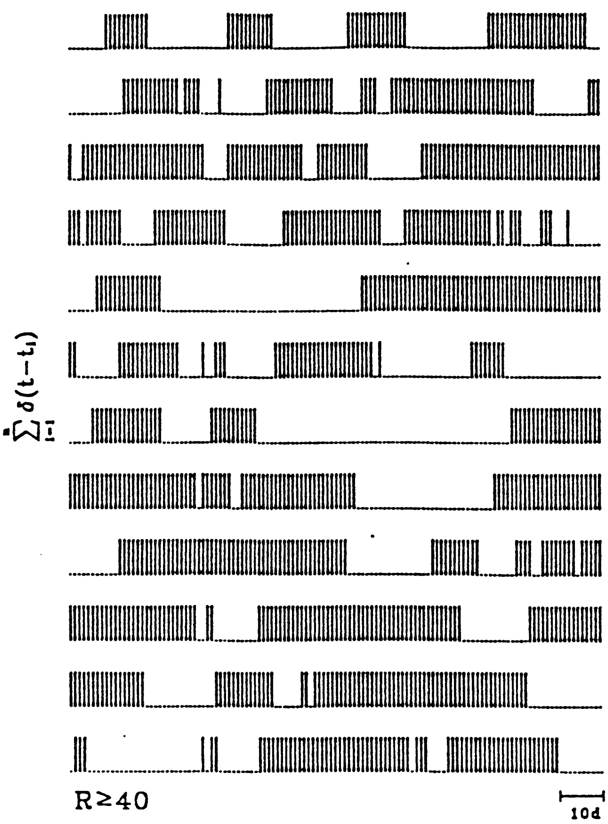

Figure 2: Occurrence of ratings of the subjective well-being with $R(t_i) \geq 40$ of the entire data set shown in Fig.1. The corresponding days are marked by Dirac's delta functions $\delta(t - t_i)$.

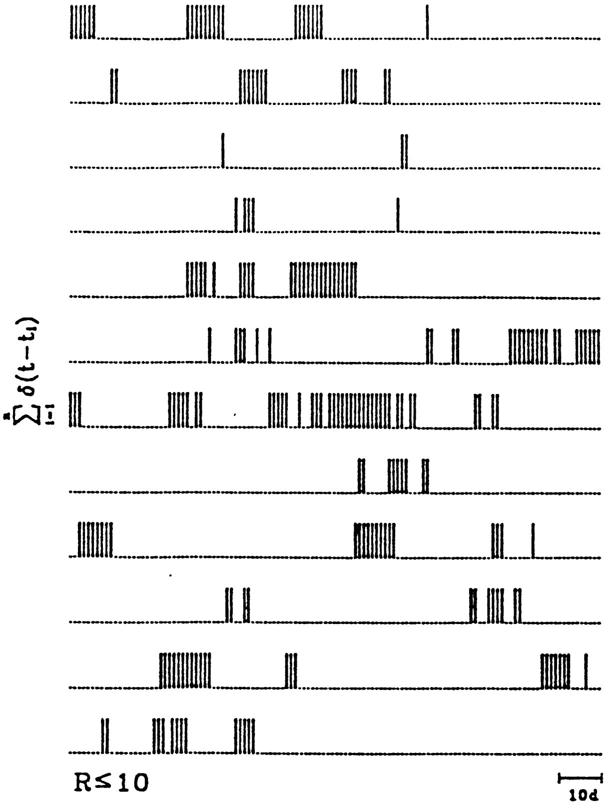

Figure 3: Occurrence of ratings of the subjective well-being with $R(t_i) \leq 10$ of the entire data set shown in Fig.1. The corresponding days are marked by Dirac's delta functions $\delta(t - t_i)$.

data, that the state of subjective well-being is not constant but fluctuates in general from day to day. By no means these fluctuations taken as a whole are simple oscillations describable by a sine function as it is assumed by performing the so-called biorhythm analysis [1]. In a rough approximation the data look as if the basic underlying mechanism responsible for the subjective well-being is a two-state ('up'-'down') system with a certain endogenic dynamics.

By introducing discriminating rating levels for the occurrence of $R(t_i) \geq r$ to reveal the fluctuations in the 'ups', the data shown in Fig. 1 were transformed into a point process. To obtain Fig. 2 the discriminating rating level was set to $r = 40$, i.e. the point process shows the fluctuations of the very strong 'ups' of the subjective well-being irrespective of their actual rating. Similar point processes for other discriminating rating levels were obtained and analyzed. In particular for determining the fluctuations in the occurrence of the very strong 'downs' $s = 10$ was chosen (Fig. 3).

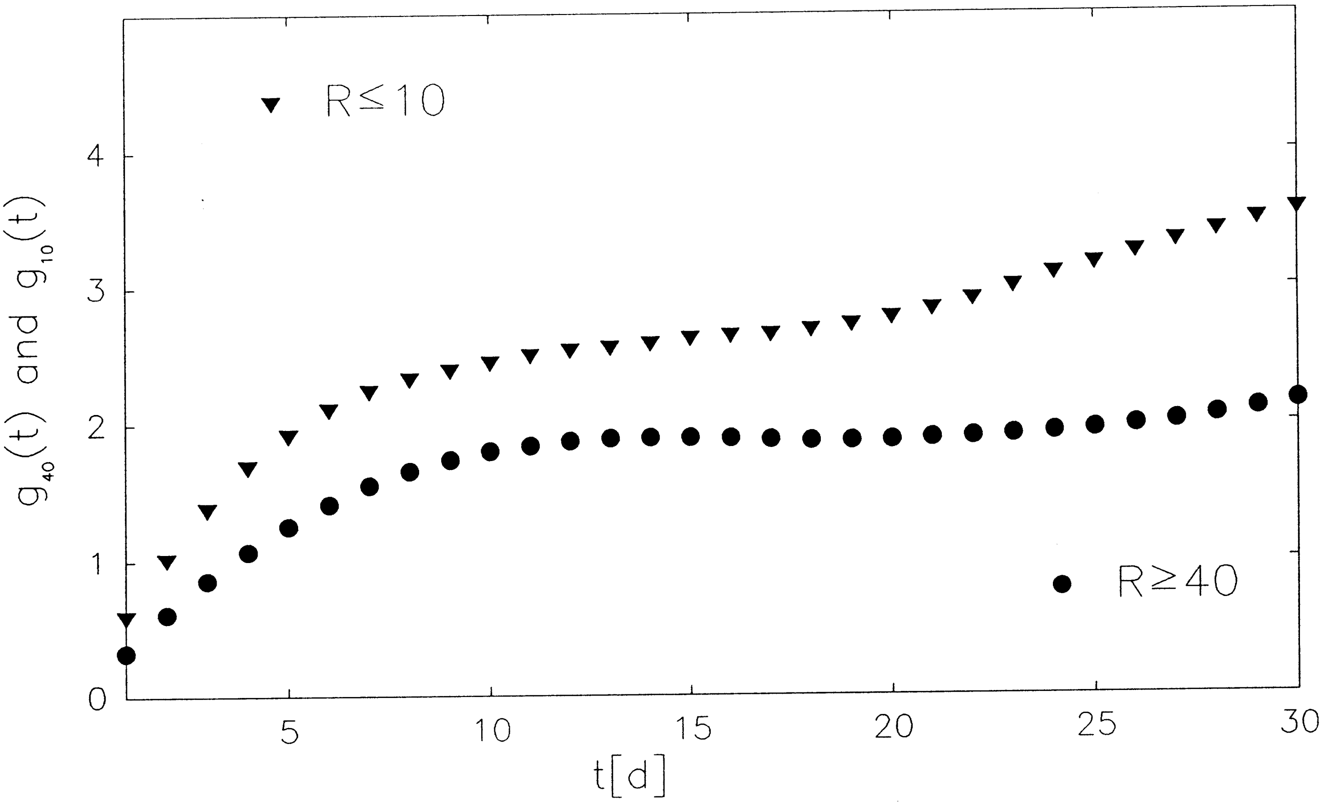

Figure 4: The clustering functions $g_{40}(t)$ and $g_{10}(t)$ of the data sets shown in Fig.2 and Fig.3 obtained from the original data shown in Fig.1 by introducing discriminating levels $r = 40$ and $s = 10$.

For all discriminating rating levels r or s the resulting point processes exhibited a certain clustering of events individually described as $\delta(t - t_i)$ (cf. Figs. 2 and 3). In order to characterize the clustering more precisely we used a clustering function $g(t)$ which was defined for earthquakes [23] as follows:

$$g_{r,s}(t) = \langle n(t) \rangle_{t_i} - t\overline{n}, \tag{7}$$

where $\langle n(t) \rangle_{t_i}$ is the number of events in the interval $(t_i, t_i + t]$ averaged over all t_i in the temporal sequence $\mathbf{S}_r = \{R(t_i) \in \mathbf{S} \mid R(t_i) \geq r\}$ or $\mathbf{S}_s = \{R(t_i) \in \mathbf{S} \mid R(t_i) \leq s\}$ and $\mathbf{S} = \{R(t_i)\}$ describing the total temporal sequence (cf. Fig. 1). $\overline{n}$ is the average density of events, i.e. the number of events within $\mathbf{S}_r$ or $\mathbf{S}_s$ divided by the total observation time. The clustering function $g_{r,s}(t)$ should measure the expected different clustering for the 'ups' and 'downs' inherent in the data set shown in Fig. 1. Examples for $g_{r,s}(t)$ are given in Fig. 4 as $g_{40}(t)$ and $g_{10}(t)$. In general, for all discriminating levels r and s analyzed, $g_{r,s}(t)$ is positive and non-decreasing, contrary to a homogeneous Poisson point process for which $g(t) = 0$ for all t. Moreover, $g_s(t) \geq g_r(t)$ for $t \leq 30$ days.

After introducing a certain r or s, for the resulting point process $\mathbf{S}_r$ or $\mathbf{S}_s$ the low frequency part of the corresponding spectrum $S(f)$ was determined by using counting statistics as described in Methods. Fig. 5 shows the result of the point processes shown in Figs. 2 and 3, i.e. the $Var[N(\Delta t)]$ for $\mathbf{S}_{40}$ and $\mathbf{S}_{10}$ are plotted on a log-log scale versus the counting windows Δt. From the straight lines fitted to the data points it is demonstrated that the variance-time curves follow the power laws

$$Var[N(\Delta t)] \propto (\Delta t)^{1+0.76} \text{ for } \mathbf{S}_{40} \tag{8}$$

and

$$Var[N(\Delta t)] \propto (\Delta t)^{1+0.62} \text{ for } \mathbf{S}_{10} \tag{9}$$

within the first scaling region $1d \leq \Delta t \leq 15d$ and thus the low frequency part of the spectrum scales as

$$S(f) \propto f^{-0.76} \text{ for } \mathbf{S}_{40} \tag{10}$$

and

$$S(f) \propto f^{-0.62} \text{ for } \mathbf{S}_{10}. \tag{11}$$

For $\Delta t > 15d$ a second scaling region was observed showing an almost random behavior: $b = 0.00$ for $\mathbf{S}_{40}$; $b = 0.08$ for $\mathbf{S}_{10}$.

Similar results, i.e. similar scaling behavior for the variance-time curve and for the spectrum were obtained for other discriminating levels r and s.

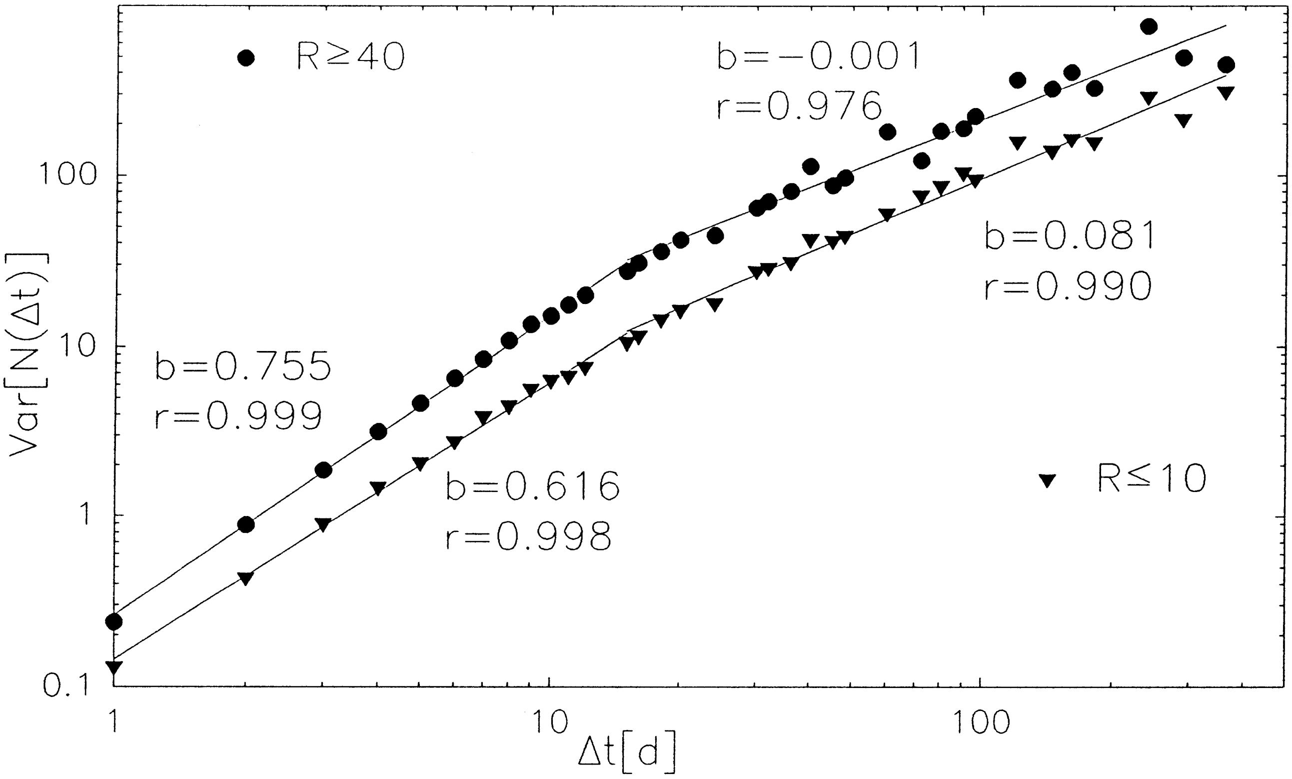

Figure 5: The variance-time curve $Var[N(\Delta t)]$ for the very strong 'ups' ($R(t_i) \geq 40$; cf. Fig. 2) and very strong 'downs' ($R(t_i) \leq 10$; cf. Fig.3) plotted on a log-log scale versus the counting window Δt. The variance-time curves scale as $(\Delta t)^{1+b}$ for $1d \leq \Delta t \leq 15d$ with indicated values of b and the corresponding correlation coefficients r. For the second scaling region, i.e. for $\Delta t \geq 15d$ the exponent $b \approx 0.00$ indicates random fluctuations of the ratings.

4 Discussion

Recently, also due to the introduction of the concept of self-organized criticality by Bak, Tang and Wiesenfeld [4], attention has been drawn to the characterization of temporal fluctuations in a number of physical and biological systems. In the following the discussion will be focused on the fluctuations of endogenous biological rhythms.

The human heart rate, even in the healthy resting subject, displays a considerable amount of fluctuations, which have been characterized as $1/f$-fluctuations [16], [24], [28], [12], [5]. Furthermore, it was demonstrated that the heart rate variability of healthy men shows periods of $1/f$-fluctuations with interpolated periods of white noise within 24 hours [20], [19].

In animal experiments it has been demonstrated that the fluctuations in respiratory intervals also exhibited $1/f$-fluctuations, but these characteristic types of fluctuation disappeared into white noise fluctuations when the end-tidal p_{CO_2} was raised to 50 or 60 mmHg [11].

The fluctuating insulin requirements of an unstable diabetic over an eight-year period have been subjected to spectral analysis and it was demonstrated that the low frequency part of the spectrum did exhibit $1/f$ characteristics [6].

Recently, the spectral analysis of the discharge of neurones located in the mesencephalic reticular formation during paradoxical sleep of the cat has revealed that in this state of the animal $1/f$-fluctuations of the neuronal discharge do exist. However, the low frequency spectral profile became flat, i.e. white noise was found during slow-wave sleep [27], [9]. So far, also the thalamic neuronal discharge exhibited $1/f$-fluctuations in the absence of intentional stimulation, but we have not seen the transition into white noise fluctuations [15], [21]. Earlier, even for the discharges in primary afferent auditory fibres $1/f$ characeristics have been reported [26].

It is tempting to speculate that the basic mechanisms underlying the neuronal and humoral activity in the central nervous system responsible for the subjective state of well-being in the absence of intentional stimulation are expressions of a self-organized critical state, as introduced by Bak, Tang and Wiesenfeld [4] for physical systems. Self-organized criticality (SOC) describes the tendency of dissipative systems with many degrees of freedom to drive themselves to a critical state with a wide range of length and time scales without any fine-tuning of external fields. The idea complements the concept of chaos, wherein simple systems with a small number of degrees of freedom can display quite complex behavior [7].

Currently, it is hard to give a rigorous definition for SOC, however, usually one gives this name to those systems which do not need fine-tuning of external fields to give power-law characteristics for the parameters describing the system. The canonical example of SOC is the cellular automaton model called 'sandpile model' introduced by Bak, Tang and Wiesenfeld [4]. The critical state is characterized by 'avalanches' (activity) with power-law spatial and temporal distribution functions limited only by the size of the system.

We assume that the subjective well-being dynamics can be described as a self-organized critical process and characterize the temporal fluctuations by its low frequency part of the power spectrum. The method applied reliably discriminates f^{-b} fluctuations with $b = 0.76$ for $\mathbf{S}_{40}$ and $b = 0.62$ for $\mathbf{S}_{10}$ in our case in the first scaling region (cf. Fig. 5) from a random point process, which would result in $b = 0.0$ as found in the second scaling region $\Delta t > 15d$.

If the neuronal/humoral system responsible for the subjective well-being is indeed operating at a self-organized critical state, an external perturbation can create either a small effect or a large one. There is in principle no limit on how long the effect may last. The degree of unpredictability is actually less severe than for chaotic systems; SOC systems are operating at the 'border of chaos' [2]. In SOC systems due to an external perturbation the maximum predictability decays as a power law, t^{-a}, where a is some constant [3]. Fluctuations due to external stimulation are much stronger in SOC systems than those being realized in an equilibrium system and can not be prevented. In case of the described biorhythm this would mean that a transition from the 'up' state to the 'down' state due to a severe external perturbation is inevitable for the individual.

As a very rough approximation the biorhythm displayed in Fig. 1 may be described by a two-state, i.e., an 'up'-'down' system with an intrinsic dynamics. Currently, with the limited amount of data it is impossible to decide, whether the neuronal/humoral system responsible for the biorhythm is a representation of a general process which has been studied under the name *stochastic resonance* [22], [17] or is the realization of an *alternating fractal renewal process* [18].

The results of the present study with an extended data set confirm those of the pilot study with only two years of monitoring $R(t_i)$ [14]. The most important result is the finding of a first scaling region $1d \leq \Delta t \leq 15d$ for various discriminating levels r and s describing the 'ups' and 'downs' when $S(f)$ is decaying as f^{-b} with $b \approx 0.7$. Therefore, based on one's own monitored biorhythm for a given time period it should be possible to predict future episodes with a certain probability by applying methods of nonlinear time series analysis [25] or modified feed-forward neural networks learning with the backpropagation algorithm.

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