Modelling and Performance Evaluation of ATM Technology (C-15)
H. Perros, G. Pujolle and Y. Takahashi (Editors)
Elsevier Science Publishers B.V. (North-Holland)

An Analysis of Trunk Reservation and Grade of Service Balancing Mechanisms in Multiservice Broadband Networks

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Abstract

We consider a broadband system carrying different types of services. The transmission system is built by a set of links having a fixed or quasi-stationary bandwidth, e.g. in the form of virtual paths arising in Asynchronous Transfer Mode networks. The bandwidth usage by different services of various bit-rates is organized in a complete sharing mode. We devote attention to two different call admission schemes: i) regular complete sharing and ii) complete sharing in conjunction with a trunk reservation mechanism. First, an overview on analytical methods dealing with models of both call admission schemes is given. An approximate recursive algorithm for trunk reservation which gives us the ability to deal with a large number of different traffic classes is described. Considering a number of services with different grade of service in term of blocking probabilities, a general rule is formulated giving the ability to equalize the call blocking probabilities of any set of arbitrary chosen service classes. Simulation results are presented to show the accuracy of the approximation. Some numerical examples are also carried out to illustrate the efficiency and the grade of service management capability of trunk reservation mechanisms.

Keyword Codes: C.4; *
Keywords: Performance of Systems; Network Operation and Management

1 Modeling of multi-service broadband systems

We consider a broadband system transmission link with a fixed transmission speed C measured in $Mbps$. A number N of different traffic classes uses the link at call level in a complete sharing mode. The arrival traffic of service class i is assumed to follow a Poisson process with rate λ_i. During the holding time T_{H_i} of a call of type i, the call generates a constant bit-rate denoted by C_i. The basic system environment is shown in Fig.1. For the analysis part the call holding time T_{H_i} is assumed to have a negative-exponential distribution function with mean $1/\mu_i$, whereas in some special cases indicated below the call holding time can have a general distribution function.

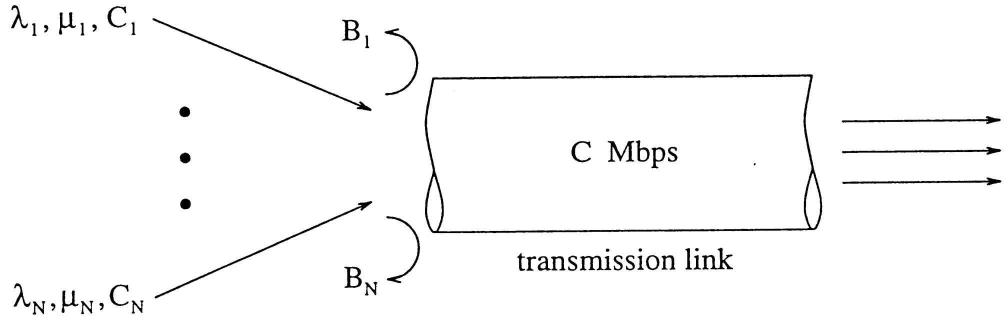

Figure 1: *Basic link model with complete sharing*

From network management viewpoint we distinguish between two call acceptance strategies:

1. *normal mode*: Let C_R be the available capacity of the link upon arrival of a call of type i. This call will be blocked if $C_R < C_i$, otherwise accepted. The resulting blocking probability for calls of type i is B_i. We will refer to this mode as call acceptance mode without trunk reservation.

2. *trunk reservation mode*: Normally a call with higher bit-rate requirement will be blocked with larger probability. To provide a capability to manage the grade of service (or to achieve a desired level of fairness) the so called trunk reservation mechanism is often discussed (cf. [9], [15], [16]). We denote the trunk reservation threshold assigned to class i by Θ_i. According to the call acceptance mode with trunk reservation, a call of type i will be blocked if $C_R < C - \Theta_i$, otherwise accepted. In other words, trunk reservation is a mechanism of bandwidth prereservation for traffic classes with higher bit-rate requirement, in order to enable bandwidth management. This results in a reduction of call acceptance of lower bit-rate call classes.

   In this context, we will formulate in a later subsection of this paper a rule to equalize the blocking probabilities of an arbitrary number of traffic classes (out of the given N classes) by setting appropriate trunk reservation thresholds.

In Enomoto et al. [6] a product form solution for the state probabilities was given for the traffic model without trunk reservation. Moreover, the case of a finite number of traffic sources was treated and the state probabilities have been also given in a product form. In Aein [1] it was shown that the product form solution is valid for all "coordinate convex policies". It is obvious that the numerical evaluation of these product forms will become numerically intractable for larger number of traffic classes. A simpler recursive solution for the state probabilities of the traffic model described above was proposed by Kaufman [10] and Roberts [14] independently of each other. In [10] the assumption upon the holding time was extended from exponential distributions to distributions with rational Laplace transform and it was shown that the restriction on "coordinate convex policies" is not necessary for the product form solution. A general distribution for the holding time was assumed in [14]. Using the recursive solution, also larger and more realistic numbers of traffic classes could be investigated without running into numerical difficulties, in contrast to the product form solution. In Barberis [2] "intermediate sharing" as generalization of complete and partial sharing has been investigated. A more general arrival process (Bernoulli-Poisson-Pascal) was considered in Delbrouck [4], where a peakedness factor was introduced. In Roberts [15] an approximate recursive solution for the state probabilities of the traffic model with trunk reservation was proposed. It was stated how to choose the threshold Θ_i to minimize the maximum blocking probability for all traffic classes. Using the results of [10], [14], and [15] it was shown in Johnson [9] and Virtamo [16] that complete sharing performs better than partial sharing. In [16] the transmission link bit-rate was calculated in dependency of B_i and C_i by a numerical approximation. An approximation of blocking probability for two traffic classes where the traffic of one class is a peakedness factor assigned and trunk reservation is employed was derived in Lindberger [12]. A slight generalization of the results in [4] and an approximate calculation of the congestion probability was given in [5] for a traffic model in which a call needs bandwidths from several parts of the transmission link. Another algorithm for the computation of the state probabilities when the arrival process is a state dependent Poisson process and the holding times are generally distributed was given in [8]. In [7] trunk reservation was investigated by an approximate iterative algorithm for the blocking probability evaluation. A general cost function for grade of service management was introduced and an iterative algorithm for the minimization of this cost function was proposed. A generalization of partial sharing where a service needs bandwidth of more than one part of the transmission link was considered in [13]. In [11] it is assumed that blocked calls can retry immediately to be admitted to the system with a smaller bit-rate requirement. To fulfill there total bit-rate requirement, the product of bit-rate and holding time is kept constant, i.e. the holding time increases proportional to the bit-rate decrease. The results are approximate and based on the recursive algorithm proposed in [10] and [14].

In Section 2.3 we present a simple rule for the balancing of blocking probabilities of different traffic classes by choosing the trunk reservation threshold Θ_i appropriately. This rule is more general than the one proposed in [15]. It turns out that balancing the blocking probabilities for different traffic classes decreases the total link utilization. On the other hand, the proportion of transmission link utilization by class-i calls is almost the same for all call classes which are subject to trunk reservation. It is also shown that the type of the call holding time distribution function does not significantly influence the blocking probability.

2 Analysis

In this section we present different analytical approaches to derive the state probabilities and related performance measures such as blocking probability and link utilization of the traffic model described in the previous section. Again the two call acceptance modes are considered:

1. *normal mode*: When trunk reservation is not employed, two ways to obtain exact blocking probabilities exist:

   i) via state probabilities using product form solution such as described in [6]

   ii) using a recursive solution according to the algorithm proposed in [10] and [14].

2. *trunk reservation mode*: In the case of trunk reservation no product form solution exists. The following possibilities exist:

   i) in principle an exact solution by solving the whole set of state equations can be obtained. From numerical point of view this method can only be evaluated for cases with small number of traffic classes; for realistic parameter sets this possibility is rather numerically intractable.

   ii) a recursive solution as indicated in [15] which is of approximate nature. In turn, it allows also the investigation of a large number N of traffic classes.

In the following we will give a brief description of the methods mentioned.

2.1 System without trunk reservation

As discussed above the system without trunk reservation mechanism can be investigated by means of a product form solution. Since the multi-dimensional state space can be mapped into a one-dimensional state space without influencing the resulting blocking probability, a recursion (cf. [10], [15]) can be used to reduce the numerical complexity. The two methods will be briefly described.

2.1.1 Product form solution

The system state is defined by the number of accepted calls from each class $(n_1, ..., n_N)$. The multi-dimensional state space (cf. Fig. 2). has thus as much dimensions as the number of call classes.

First, the unnormalized state probabilities $\tilde{p}(n_1, ..., n_N)$ can be given as:

$$\tilde{p}(n_1, ..., n_N) = \begin{cases} \prod_{i=1}^{N} \frac{(\lambda_i/\mu_i)^{n_i}}{n_i!} & : \sum_{i=1}^{N} n_i C_i \leq C \\ 0 & : \sum_{i=1}^{N} n_i C_i > C \end{cases} . \qquad (1)$$

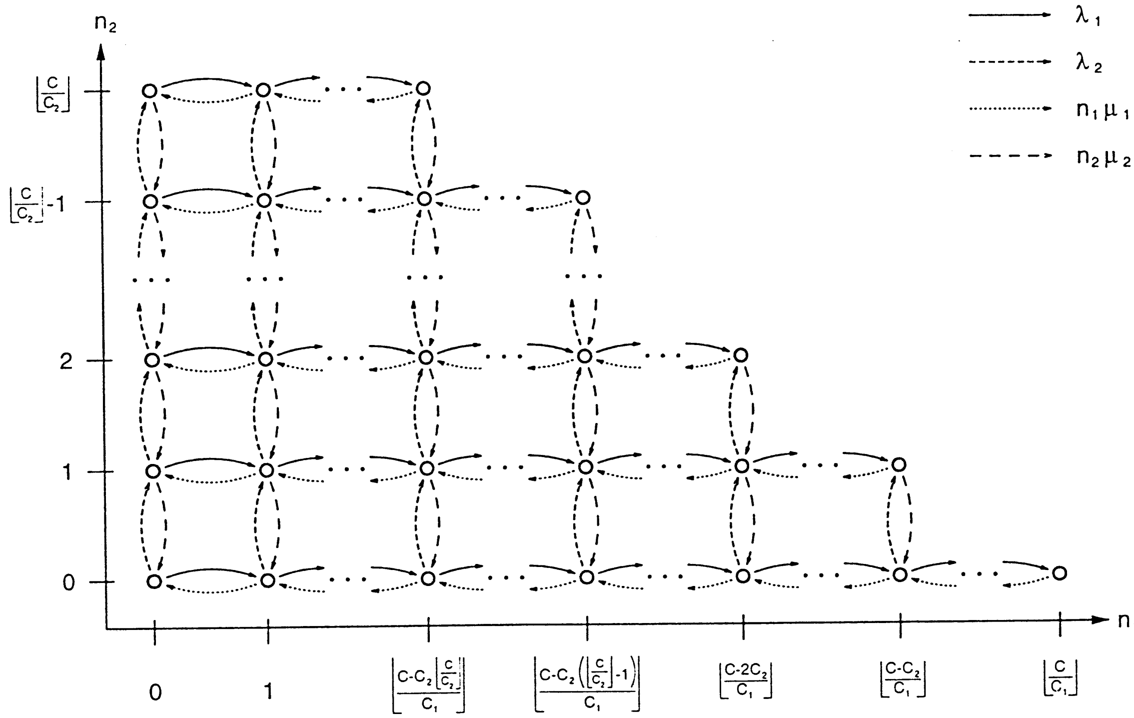

Figure 2: *State transition diagram for product form solution.*

We arrive then at the state probabilities by normalization:[1]

$$p(n_1, ..., n_N) = \tilde{p}(n_1, ..., n_N) \cdot \left(\sum_{n_1=0}^{\lfloor C/C_1 \rfloor} \cdots \sum_{n_N=0}^{\lfloor C/C_N \rfloor} \tilde{p}(n_1, ..., n_N) \right)^{-1} . \tag{2}$$

With these state probabilities the probability of blocking an arriving call of class i is:

$$B_i = \sum_{(n_1,...,n_N) \in S_i} p(n_1, ..., n_N) \tag{3}$$

where S_i is defined by:

$$S_i = \{(n_1, ..., n_i, ..., n_N) \mid (n_i + 1)C_i + \sum_{\substack{j=1 \\ j \neq i}}^{N} n_j C_j > C\}. \tag{4}$$

[1] $\lfloor x \rfloor$ denotes the largest integer less or equal x

2.1.2 Recursive solution

The recursive solution of the model is based on a mapping of the multi-dimensional state space to a one dimensional state space (cf. [10], [14]), in accordance with a proper bandwidth discretization. A basic bandwidth unit $\triangle C$ is therefore defined by:[2]

$$\triangle C = gcd\{C_i\} \qquad\qquad 1 \le i \le N. \tag{5}$$

In communication system environments, this basic bandwidth unit could be e.g. 64 kbps or 2.048 Mbps. The maximum number of available basic bandwidth units is denoted by $M = \lfloor C/\triangle C \rfloor$ and the number of required basic bandwidths per class-i call by $m_i = \lfloor C_i/\triangle C \rfloor$. The state of the traffic model is defined by the number m of occupied basic bandwidth units. In Fig.3 we show the resulting state space after the mapping. The bandwidth of class-1 calls C_1 is assumed as $\triangle C$ (this results in $m_1 = 1$).

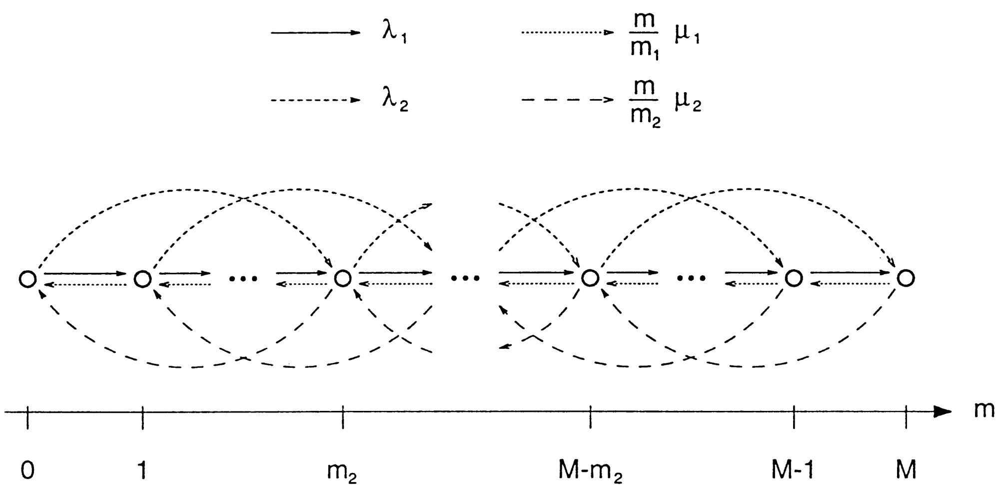

Figure 3: *State space reduction for the recursive solution.*

The unnormalized state probabilities can be derived by the following recursion algorithm:

$$\tilde{p}(m) = \begin{cases} 1 & : \text{ for } m = 0 \\ 0 & : \text{ for } m < 0 \\ \frac{1}{m} \sum_{i=1}^{N} \tilde{p}(m - m_i) m_i \frac{\lambda_i}{\mu_i} & : \text{ for } 0 < m \le M \end{cases} . \tag{6}$$

By normalization we get the state probabilities $p(m)$:

$$p(m) = \tilde{p}(m) \cdot \left(\sum_{m=0}^{M} \tilde{p}(m) \right)^{-1} . \tag{7}$$

[2] gcd means the greatest common divisor

The blocking probability B_i for class-i calls can be calculated by:

$$B_i = \sum_{m=M-m_i+1}^{M} p(m). \tag{8}$$

It should be mentioned here that regarding the blocking probability calculation the recursive solution discussed in this subsection is exact [10], [14]. Furthermore it has been shown in [14] that the solution delivers exact results also for general holding time distributions.

2.2 System with trunk reservation

While considering the case of trunk reservation, we are leaving the area of state spaces with product form solution. In principle, at least for smaller state spaces, the state equation systems can completely be formulated and resolved using e.g. an iterative algorithm. This possibility is however numerically intractable for more realistic parameters with larger state space. This motivates the necessity of an approximative solution.

The recursive technique used in this paper is again based on a mapping of the multi-dimensional state space into a one-dimensional state space, as discussed above. The use of this technique for this class of model was first proposed in [15].

When employing trunk reservation, a call of class i is accepted only when at most m_i basic bandwidth units are available upon arrival *and* not more than Θ_i Mbps are occupied. When calls from a class j will not be subject to trunk reservation Θ_j is set to C. To derive a recursive solution for the state probabilities, the state of the traffic model is defined as in Subsection 2.1.2 by the number m of occupied basic bandwidth units. It should be stated here that this state space description is of approximate nature. The state space from Fig.3 reduces to the state space shown in Fig.4. In difference to Fig.3 state transitions for class-1 calls are omitted if more than $M - m_2$ basic bandwidth units are occupied. This is due to the fact that a class-1 arrival is blocked if more than $M - m_2$ basic bandwidth units are occupied although the transmission link would be able to carry one more call of this class. As a consequence the blocking probability for the class-2 calls is smaller due to the approximation.

To emphasize the approximation we denote the state probabilities of the one-dimensional state space by $p^*(m)$ and the resulting blocking probability obtained by the recursive solution by B_i^*.

The unnormalized state probabilities can be approximately obtained using the following recursion algorithm (see [9], [15]) which is similar to the one given by eqn.(6):

$$\tilde{p}^*(m) = \begin{cases} 1 & : \text{ for } m = 0 \\ 0 & : \text{ for } m < 0 \\ \frac{1}{m} \sum_{i=1}^{N} \tilde{p}^*(m - m_i) m_i(m) \frac{\lambda_i}{\mu_i} & : \text{ for } 0 < m \leq M \end{cases} \tag{9}$$

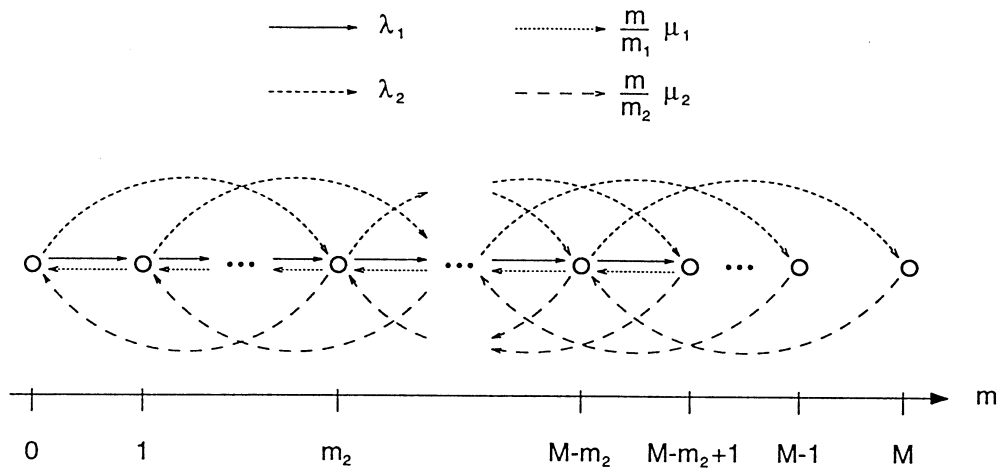

Figure 4: *Trunk reservation mode: approximate state space.*

where $m_i(m)$ is defined by:

$$m_i(m) = \begin{cases} m_i & : \quad m \cdot \triangle C \leq \Theta_i \\ 0 & : \quad m \cdot \triangle C > \Theta_i \end{cases} . \tag{10}$$

After a normalization of $\tilde{p}^*(m)$ we arrive at the state probabilities $p^*(m)$:

$$p^*(m) = \tilde{p}^*(m) \cdot \left(\sum_{m=0}^{M} \tilde{p}^*(m) \right)^{-1} . \tag{11}$$

The blocking probability B_i^* for calls of class i can be calculated by:

$$B_i^* = \sum_{m=\min\{M-m_i, \Theta_i/\triangle C\}+1}^{M} p^*(m). \tag{12}$$

2.3 Blocking probability equalization rule

In general, calls with higher bit-rate demand C_i have higher blocking probability than calls with lower bit-rate demand C_j ($B_i > B_j$). The aim of trunk reservation is to influence performance parameters as e.g. blocking probability of different traffic classes. Out of the number N of service classes we may want to influence the grade of service in such a way that blocking probabilities of certain classes of services should be dimensioned to be the same. This will be done by choosing appropriate values of the trunk reservation thresholds.

We propose the following simple and general rule for balancing blocking probabilities of calls from different classes:

Rule: To force $B_i = B_j = ... = B_k$ for arbitrary $i, j, ..., k$, set the corresponding thresholds $\Theta_i, \Theta_j, ..., \Theta_k$ to $\lfloor (C - \max\{C_i, C_j, ..., C_k\}) \rfloor$.

Note that the equalization of the B_i's implies also the equalization of the B_i^*'s. This rule is more general than the rule proposed in [15]. To motivate the rule we refer to Fig.3. Blocking probabilities are equal if the sets of states leading to call blocking are the same for each of the classes which are subject to trunk reservation. As we will discuss later in the paper using numerical examples, a side effect of having trunk reservation or blocking probability equalization is that also a fairer link utilization is obtained (i.e. the class-i link utilization is proportional to the normalized offered bit-rate traffic of class-i calls).

3 Numerical examples

In this section numerical results will be presented to show two main effects: i) the approximation accuracy of the recursion algorithm and ii) the efficiency of the trunk reservation mechanism and in accordance, the blocking probability and the grade of service management issues. Since the traffic regulation is done on call level while the link occupancy by different call service classes is measured on bit (or cell) level, we will first introduce some notations concerning offered traffic, carried traffic and link utilization on call and bit-rate levels. While on *call level* the offered traffic A_i and the carried traffic Y_i of class i are given by:

$$A_i = \frac{\lambda_i}{\mu_i} \qquad Y_i = A_i \cdot (1 - B_i), \tag{13}$$

we obtain on the *bit-rate level* the normalized offered bit-rate traffic α_i as

$$\alpha_i = \frac{\lambda_i}{\mu_i} \cdot \frac{C_i}{C} = A_i \cdot \frac{C_i}{C} \tag{14}$$

and the normalized carried bit-rate traffic ρ_i of class-i calls as

$$\rho_i = \frac{\lambda_i}{\mu_i} \cdot \frac{C_i}{C} \cdot (1 - B_i) = \alpha_i \cdot (1 - B_i). \tag{15}$$

Note that ρ_i also represents the link utilization by class-i calls. Finally, the total link utilization ρ is given by:

$$\rho = \sum_{i=1}^{N} \rho_i = \sum_{i=1}^{N} \frac{\lambda_i}{\mu_i} \cdot \frac{C_i}{C} \cdot (1 - B_i). \tag{16}$$

The first numerical example deals with the case of two traffic classes with $C_1 = 2$ Mbps and $C_2 = 20$ Mbps which offer the same offered bit-rate traffic ($\alpha_1 = \alpha_2$) to the transmission link of capacity $C = 150$ Mbps. The blocking probability and the link utilization for this set of parameters is shown in Fig.5 and Fig.6 respectively. When trunk reservation is not employed the blocking probability (see Fig.5) for class-2 calls is clearly higher than that of class-1 calls due to the higher bit-rate of class-2 calls ($C_2 > C_1$). The trunk reservation threshold Θ_1 has been chosen according to the rule stated in Section 2.3 to $\Theta_1 = 130$ Mbps and equalizes the blocking probabilities of calls from both classes. We can observe the effect that on call level the blocking probability B_1 increases much more than B_2 decreases.

The simulation results show that the accuracy of the recursive approximation is good for the whole range of traffic intensities. We have observed this level of accuracy also for a large number of other parameter sets and different traffic mixes ($\alpha_1 \neq \alpha_2$) whose results are not shown here. It can be stated here that the approximate solution for the case of trunk reservation can be generally considered as sufficient. The difference between the exact blocking probability B_i and the approximate one B_i^* is neglectable for practical purposes.

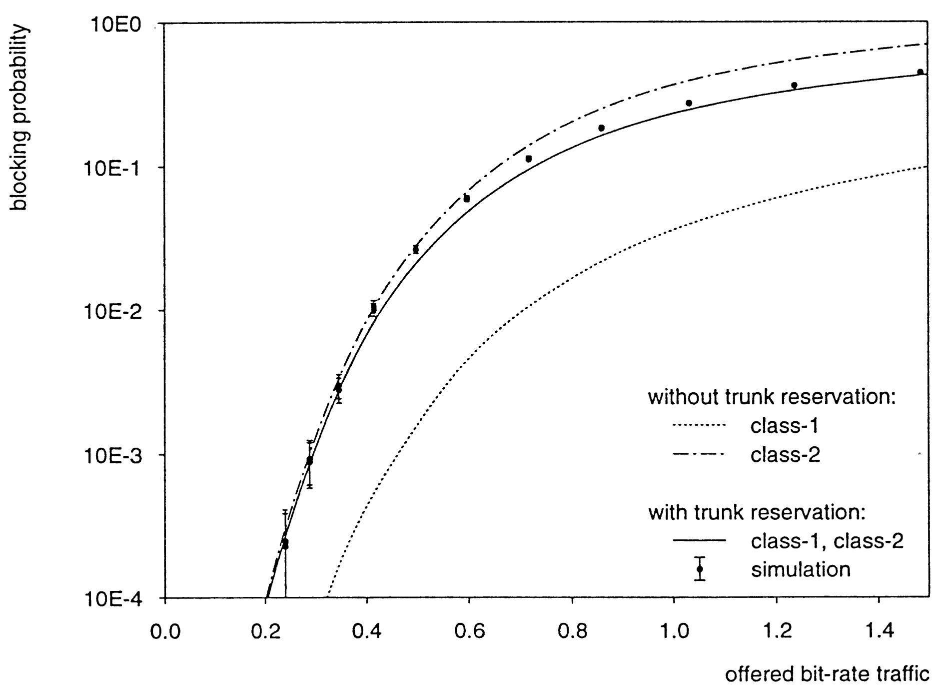

Figure 5: *Blocking probability equalization by trunk reservation and approximation accuracy.*

The transmission link utilization (cf. Fig.6) by class-1 calls ρ_1 is higher than the utilization by class-2 calls ρ_2 without trunk reservation. By employing trunk reservation the utilization from both classes is also equalized. The price to be paid for that equalization is that the total transmission link utilization ρ decreases with trunk reservation.

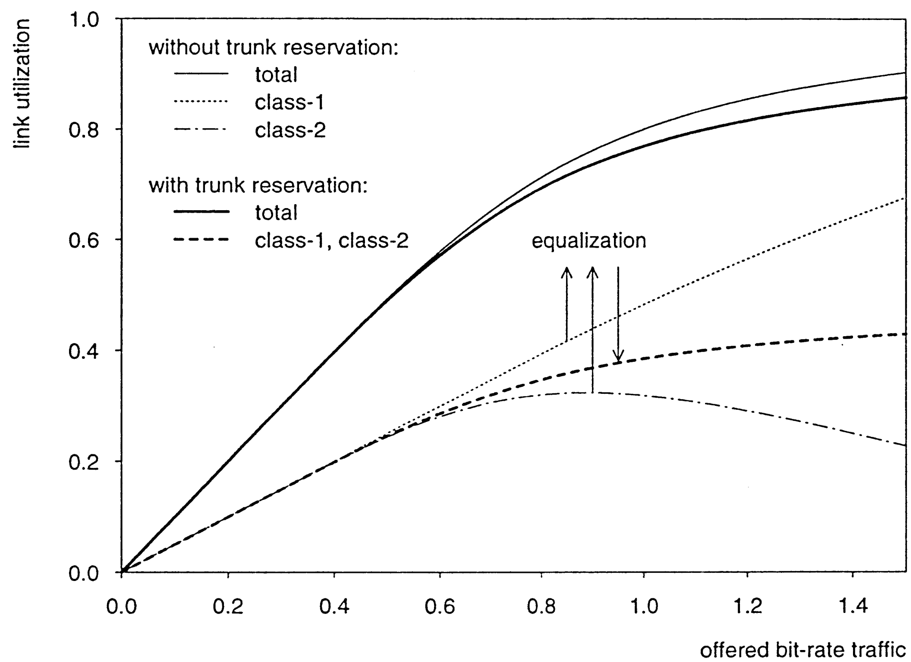

Figure 6: *Link utilization sharing with trunk reservation.*

Results for a more complex example are shown in Fig.7 and Fig.8 respectively. The transmission link has a bit-rate of $C = 150$ Mbps and we consider four traffic classes with bit-rates $C_1 = 2$ Mbps, $C_2 = 5$ Mbps, $C_3 = 10$ Mbps, $C_4 = 20$ Mbps. Calls from the different traffic classes are assumed to offer the same load ($\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$) and we want to equalize the blocking probability and transmission link utilization of class-1 and class-3 calls. Fig.7 shows that a trunk reservation threshold $\Theta_1 = 140$ Mbps not only equalize the blocking probabilities of class-1 and class-3 calls but also decrease the blocking probabilities of class-2 and class-4 calls.

As expected from the results for the blocking probability the utilization of the transmission link by class-1 and class-3 calls (ρ_1, ρ_3) is also equalized by means of the trunk reservation mechanism with $\Theta = 140$ Mbps (see Fig.8). The transmission link utilization by class-2 and class-4 calls (ρ_2, ρ_4) is increased as the blocking probability is decreased. The numerical results show that using the trunk reservation mechanism a possibility exists, which allows to equalize blocking probabilities and transmission link utilizations from calls of different traffic classes.

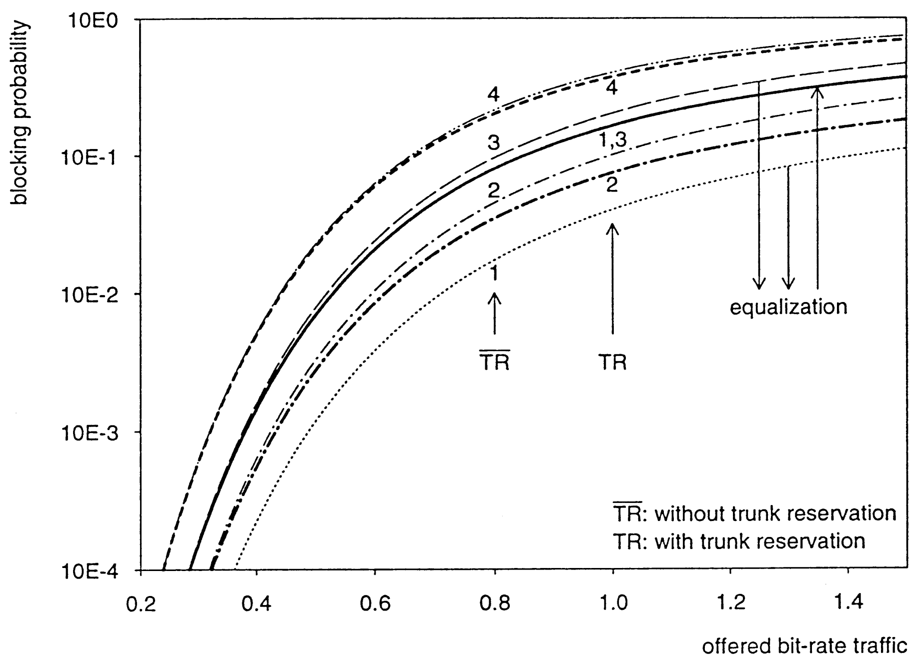

Figure 7: *Blocking probability equalization in multi-service system.*

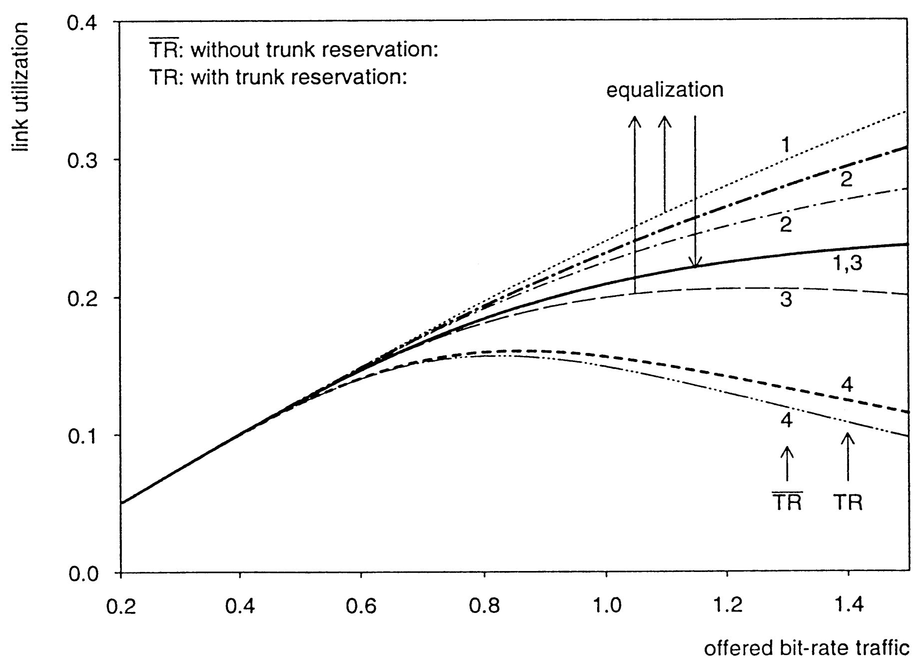

Figure 8: *Link utilization sharing in multi-service system.*

Furthermore, using simulations, we have observed that the results for the system with trunk reservation are insensitive against the holding process type as can be seen in Table 1. The blocking probabilities of the two call classes are shown in dependence on the coefficient of variation c_{T_H} of the holding time distribution which varies from 0 (Deterministic holding process), 1/3 (Erlang-9), 1 (Poisson), to 4 (Hyperexponential). It can be observed that the blocking probabilities of the two call classes are almost the same no matter what kind of holding process type is used.

	$c_{T_{H_1}} = c_{T_{H_2}} = 0$		$c_{T_{H_1}} = c_{T_{H_2}} = 1/3$		$c_{T_{H_1}} = c_{T_{H_2}} = 1$		$c_{T_{H_1}} = c_{T_{H_2}} = 4$	
α	class-1	class-2	class-1	class-2	class-1	class-2	class-1	class-2
0.3	1.28E-3	1.34E-3	1.28E-3	1.32E-3	1.27E-3	1.29E-3	1.27E-3	1.28E-3
0.4	8.10E-3	8.06E-3	8.05E-3	8.05E-3	7.90E-3	7.96E-3	7.80E-3	7.74E-3
0.5	2.68E-2	2.69E-2	2.68E-2	2.68E-2	2.66E-2	2.65E-2	2.56E-2	2.57E-2
0.6	5.97E-2	5.95E-2	5.95E-2	5.95E-2	5.94E-2	5.92E-2	5.73E-2	5.73E-2
0.7	1.05E-1	1.05E-1	1.05E-1	1.05E-1	1.04E-1	1.04E-1	1.00E-1	1.00E-1
0.8	1.56E-1	1.56E-1	1.56E-1	1.56E-1	1.55E-1	1.55E-1	1.51E-1	1.51E-1
0.9	2.08E-1	2.09E-1	2.08E-1	2.09E-1	2.07E-1	2.07E-1	2.03E-1	2.03E-1
1.0	2.58E-1	2.58E-1	2.58E-1	2.58E-1	2.57E-1	2.57E-1	2.53E-1	2.53E-1
1.1	3.04E-1	3.05E-1	3.04E-1	3.04E-1	3.03E-1	3.03E-1	2.99E-1	2.99E-1
1.2	3.47E-1	3.47E-1	3.47E-1	3.47E-1	3.45E-1	3.45E-1	3.44E-1	3.44E-1

Table 1: *Illustration of trunk reservation mechanism robustness against holding process type* ($\alpha = \alpha_1 + \alpha_2$).

4 Conclusion and outlook

In this paper we have considered a broadband system with different types of services. The transmission system is built by a set of links having a fixed bandwidth. The bandwidth usage by different services of various bit-rates is organized in a complete sharing mode. We investigated regular complete sharing and complete sharing with a trunk reservation mechanism as call admission methods. The analytical methods dealing with the related queueing models have been reviewed. An approximate recursive algorithm for trunk reservation which enables us to deal with a large number of different traffic classes was described. A general rule for the blocking probability equalization was formulated for any set of arbitrary service classes. Simulation results have been presented to show the accuracy of the recursion approximation. Some numerical examples are also carried out to illustrate the efficiency and the grade of service management capability of trunk reservation mechanisms. An extension of the investigation towards variable bit-rate call types is under study.

Acknowledgement

The authors would like to thank M. Ritter for the stimulating discussions and for the programming efforts during the course of this work.

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