An analytical approach for determining coverage probabilities in large UMTS networks

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Abstract. **An analytical model for dimensioning and planning of large UMTS-networks with varying service mixtures is presented. Given the distribution of the UMTS traffic and the attenuation between mobiles and base stations using any given path loss prediction, the uplink-coverage probabilities and call blocking rates can be determined for a realistic "many cells-many mobiles" case. Because of effects like "cell breathing" a precise calculation of in-cell and inter-cell interference is done simultaneously. Imperfection of power control is taken into account. This analytical model permits fast calculations in particular for large networks.**

I. INTRODUCTION

The dimensioning and planning of large UMTS-networks requires tools capable to cope with thousands of cells and even millions of subscribers. Since the commonly used Monte Carlo simulation technique is very time consuming, analytical approaches are required instead. Because of CDMA-networks being highly influenced by interference, a precise calculation of in-cell and inter-cell interference has to be done. Therefore algorithms should take the realistic "many cells-many mobiles"-case into account. Furthermore varying mixtures of services with different QoS constraints, the subscribers' varying speeds, different equipment, etc. have to be considered. Previous analyses of CDMA networks mostly restrict to the case of just one bearer service, see for example the work of Veeravalli and Sendonaris [1] and the extensions in [3].

To develop efficient algorithms for network planning we firstly have to give a precise mathematical model describing the mobile–base station link. In section II we explain the basic assumptions of our model and give some notations. Then we present a basic algorithm to determine coverage probabilities for the uplink. The single steps of the basic algorithm are explained in section III and IV. In section III we describe the model and outline how to compute the interference at every base station simultaneously. Given this information and the attenuation prediction from any propagation model for the link between a mobile and a base station in any place we are able to determine the needed transmission power of a mobile of any chosen bearer service and place. Additionally we compute the in-cell and inter-cell loading of all cells.

To determine the coverage probabilities for any place and bearer service in the network we have to know the distribution functions of transmission powers. In section IV a way to determine them is explained. Starting with the load caused by one subscriber at the serving base station and given the traffic distribution we show how to get distribution functions for the noise rise. Knowing them it is easy to compute the wanted function. As an application we present how to determine the coverage probability for a mobile at a given place. Our conclusions are given in section VI.

This paper is dealing with considerations for the uplink only and does not take extensions like soft and softer handover into account; the effects of soft handover are given in a second contribution to this conference [2]. An analytical approach for determining downlink coverage and capacity will be given in a forthcoming paper.

II. MODEL ASSUMPTIONS AND BASIC ALGORITHM

We assume many cells and many mobiles, each belonging to exactly one cell, called the home cell. All mobiles have an individual data rate, an activity factor and a required E_b/N_0-value at the base station. Each base station corresponds to one or, in the case of sectorization, more cells. We use the following notations.

L	the number of cells
K	the number of mobiles
l	a cell
k	a mobile
R_k	data rate of k [bps]
ν_k	activity factor of k [0..1]
ε_k^*	target-SIR of k [dB]
S_k	transmission power k [dBm]
$d_{k,l}$	attenuation from k to l [dB]
$\varepsilon_{k,l}$	the received E_b/N_0 of k at l [dB]
W	chip rate [Hz]
$(N_0)_l$	thermal noise power density at l [dBm]

η_l cell loading of l [0..1]

For any parameter *x* given in decibel the linear value is $\hat{x}$. The model is based on the following equations for calculating the signal-to-interference-rate (SIR) of mobile *k* and the interference power density at base station *l* respectively.

$$\hat{\varepsilon}_{k,l} = \hat{S}_k \hat{d}_{k,l} \Big(W(\hat{N}_0)_l + \sum_{\substack{j=1 \\ j \neq k}}^{K} v_j \hat{S}_j \hat{d}_{j,l} \Big)^{-1} \frac{W}{R_k} \quad (1)$$

$$\hat{I}_l = \frac{1}{W} \sum_{j=1}^{K} v_j \hat{S}_j \hat{d}_{j,l} \quad (2)$$

After introducing formal notations, we formulate the following basic algorithm to determine coverage probabilities:

1. calculate the interference at all cells;
2. calculate the transmission powers of all mobiles;
3. determine the probability function of received powers at the (sectors of the) base stations, determine from this the probability functions of transmitting powers;
4. compare the corresponding distribution functions with the maximum transmission power to specify the coverage probability for the given mobile.

These four steps are outlined in more detail in the next chapters; steps 1 and 2 are explained in section III, steps 3 and 4 in section IV.

III. Interference and Transmission Power Calculation

Calculating the interference power densities $\hat{I}_l$ as above, Equation (1) is simplified and we are able to solve it for $\hat{S}_k$ after replacing $\hat{\varepsilon}_{k,l}$ by its target value $\hat{\varepsilon}_k^*$. To calculate $\hat{S}_k$ the interference power densities at each station have to be known. For sake of convenience, we reformulate all the equations in one matrix equation

$$\hat{S} = W(\hat{N}_0 + \hat{I})\,(P./\hat{d})^T\ \beta\,(WE + \beta\tilde{v})^{-1} \quad (3)$$

where $(P./\hat{d})_{k,l} = \hat{d}_{k,l}{}^{-1}$ if l is the home cell of k and 0 otherwise. Here, $\tilde{v}$ and β are $K \times K$-matrices with the activity factors and the products $R_k \hat{\varepsilon}_k^*$ in the diagonal, respectively. Multiplying Eqn. (3) by $W^{-1}\tilde{v}\,\hat{d}$ and replacing $W^{-1}\hat{S}\,\tilde{v}\,\hat{d}$ by $\hat{I}$, we introduce matrix A and obtain Eqn. (4) and (5):

$$A = W^{-1}(P./\hat{d})^T\ \beta\,(WE + \beta\tilde{v})^{-1}\tilde{v}\,\hat{d} \quad (4)$$

$$\hat{I} = (\hat{I} + \hat{N}_0)A \quad (5)$$

Matrix A is of great importance. We call them the "quasi-load matrix". To explain this we define cell loading in a common way.

$$\eta_l = \frac{\hat{I}_l}{\hat{I}_l + (\hat{N}_0)_l} \quad (6)$$

We get (7) directly by transforming (5):

$$\eta_l = A_{l,l} + \sum_{\substack{m=1 \\ m \neq l}}^{L} \left[\frac{\hat{I}_m + (\hat{N}_0)_m}{\hat{I}_l + (\hat{N}_0)_l} \right] A_{m,l} \quad (7)$$

Therefore the elements of the main diagonal of *A* contain exactly the in-cell-loading while the other values of *A* give an estimation of the loading caused by mobiles connected to other home cells. The entries of *A* are of the following shape:

$$A_{m,l} = \sum_{i \in C(m)} \frac{\hat{d}_{i,l}}{\hat{d}_{i,m}} \frac{R_i \hat{\varepsilon}_{i,m} v_i}{W + R_i \hat{\varepsilon}_{i,m} v_i} \quad (8)$$

Here the meaning of $C(m)$ is the set of all subscribers *i* in the network with home cell *m*.

Furthermore (5) is solvable if all column sums of A are strictly less than 1, this is exactly the condition of the so called A_{out}-case in [1]. Notice that this is a sufficient but not necessary condition for solving (5). By (3) we are able to calculate the transmission powers of all mobiles once having a solution of (5) and so step 1 and 2 of our basic algorithm are done.

IV. Calculation of Distribution Functions

The model given in the above section is exact if the active subscribers and their locations are known and can therefore be used for calculations in Monte-Carlo simulators. But here the aim is an analytical approach and so we don't know the number of (active) subscribers in our network nor their place. Therefore we have to deal with probability functions. In this section we present an approach to determine the distribution function of cell loads. We assume that the inter-cell load is similar distributed than the in-cell load. To determine the exact distribution functions of inter-cell-loads is more complicated, as can been seen from Eqn (7) and (8). But this is already in progress and will be presented soon in a forthcoming paper.

For planning mobile networks it is important to know which area is covered with a sufficient quality and how large the capacity of the network is. An area is covered if a mobile is able to detect the downlink signal and to transmit its signals with sufficient power to reach the required SIR at its home base station. Because of considering the uplink only, coverage probability means the probability not to exceed the maximum transmitting power of the mobile.

Reformulating (3) the required transmission power of a mobile is given by

$$\hat{S}_k = \frac{1}{\hat{d}_{k,l}} W(\hat{N}_0)_l \frac{1}{1-\eta_l} \frac{R_k \hat{\varepsilon}_k^*}{W + R_k \hat{\varepsilon}_k^*} \qquad (9)$$

depending on the chosen bearer service, the attenuation to its home base station and the load in his home cell (resp. the noise rise: $\hat{F}_l = (1-\eta_l)^{-1}$). Attenuation and load are the two main stochastical influences of the transmission power. The attenuation is influenced by fading, which is well described as random variables, cf.[5], for example. The distribution function of cell load is also of interest for determining the cell-capacity of the uplink, which will become clear at the end of this section.

To determine the probability functions of cell loads we start with the probability mass function (pmf) of the load caused by one active subscriber in his home cell. Afterwards we are able to calculate the pmf of load caused by 2,3,... simultaneous active subscribers using the same bearer service. Using this conditioned probability mass functions we calculate the pmf of cell load caused by users of this bearer service. In a final step we convolve these pmf's to a general pmf of cell load.

Recalling (7) and (8) the in-cell load is given by

$$\eta_l^{in} = A_{l,l} = \sum_{i \in C(l)} \frac{R_i \hat{\varepsilon}_{i,l} \nu_i}{W + R_i \hat{\varepsilon}_{i,l} \nu_i}. \qquad (10)$$

Computing the distribution functions for many-service environments requires as a first step the calculation of the common distribution functions of all mobiles using the same service. We say that mobiles with equal data rates, activity factors, and target E_b/N_0-values belong to one class.

The load generated by one active mobile in its home cell is given by $(R_k \hat{\varepsilon}_{k,l})/(W + R_k \hat{\varepsilon}_{k,l})$, cf. (10). Because of the imperfection of power control, $\varepsilon_{k,l}$ fluctuates around ε_k^*. Field trials reported that the probability mass function (pmf) of $\varepsilon_{k,l}$ is well modelled as Gaussian [4], so the desired pmf is given by transformation (cf. [1]).

The conditioned probability mass function of a fix number (say *n*) of mobiles belonging to the same class is obtained by *n*-times convolution of the above given pmf; assuming that the load contributions of the mobiles are independent. This can be done numerically.

To get the pmf of the total cell load caused by the mobiles of one class one needs the probability function of the number of active subscribers. According to a Poisson process the number of subscribers in a cell is mostly assumed to be Poisson, say with mean K_l. Each subscriber *k* is assumed to be active (means: sending) with probability ν_1. Therefore the number of active subscribers of one class follows a binomial distribution. So again the number of active subscribers of one class in a cell is Poisson with mean $\nu_1 K_l$. The desired $pmf(\eta_{l,1})$ is calculated by the theorem of total probabilities:

$$pmf(\eta_{l,1}^{in}) = \sum_{k=0}^{\infty} pmf(\eta_{l,1}^{in} \,|\, \#C(l) = k)\, po(\nu_1 K_l, k) \qquad (11)$$

Convolving the pmfs of all classes yields the general pmf of the cell load.

It might be of great importance for determining coverage probabilities not to consider the above given pmf but a conditioned pmf, under the condition of at least one active subscriber. This is of great influence for subscribers using bearer services with high data rates. In this case the cell load pmf of the load caused by mobiles from the considered class has to be calculated by a modified pmf of number of active subscribers as is shown in (12).

$$\begin{aligned} &pmf(\eta_{l,1}^{in} \,|\, \text{at least one mobile of class 1}) = \\ &\frac{1}{(1-po(\nu_1 K_{l,1}, 0))} \sum_{k=1}^{\infty} pmf(\eta_{l,1} \,|\, \#C(l) = k)\, po(\nu_1 K_{l,1}, k) \end{aligned} \qquad (12)$$

Again convolution with the (non-conditioned) pmf's of other classes cell load yield the desired probability function.

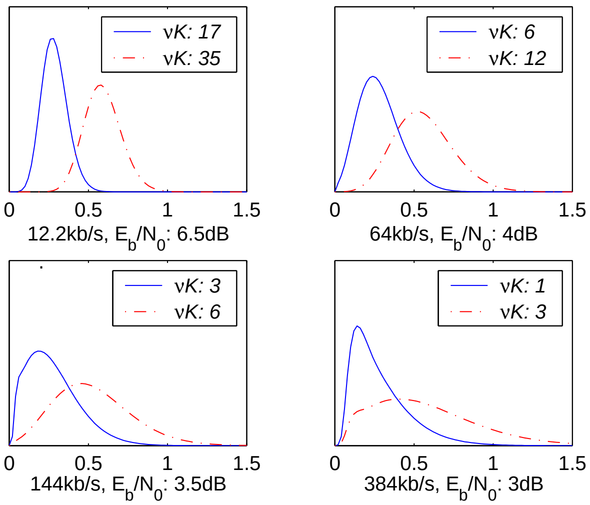

Fig. 1. Pmf of cell load for various bearer services in single services environments. νK denotes the mean of the number of active subscribers

Given the conditioned probability functions of cell load for a service one directly get the probability functions for received powers following (9) by transformation. The distribution function of transmitting powers is calculated by linear translation (after transforming in logarithmic values) of the corresponding attenuation and convolution with a Gaussian distribution modelling slow fading. The coverage probability of a mobile in a given place and of a given class is the value of the distribution function of transmitting powers at the digit of the maximal transmitting power of mobiles of this class.

Furthermore one can determine the uplink-call blocking rate from the distribution of cell load. The capacity of the uplink in UMTS-networks is bounded by a maximal threshold for cell load. Therefore the call blocking rate is exactly the value of the complementary distribution function of the cell load at the digit of the given threshold.

V. Some Numerical results

All computations have be done for a UMTS-network with chip rate 3.84MHz. We consider four bearer services with data rates 12.2kb/s, 64kb/s, 144kb/s and 384kb/s. The target SIR are set to 6.5dB, 4dB, 3.5dB and 3dB, respectively. The standard deviation of imperfect power control is assumed to be 2.5dB and the thermal noise density -169dBm/Hz.

In Figure 1 we present four probability mass functions for the cell load, one for every considered bearer service in a single service environment. In every subplot two graphs are given, for different means of the number of expected active subscribers of that bearer service. The first one is nearly a fourth, the second a half of the theoretical pole capacity in a cell (cf. [6]). If bearer services with high data rates are taken into account the assumption of log-normal distribution for cell loads will no longer be acceptable, although for voice services (data rate: 12.2 kb/s) this seems to be a good approximation (cf. [1]).

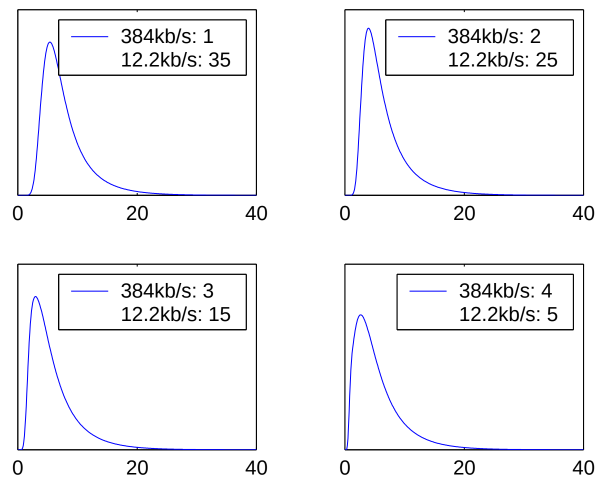

Fig. 2. Pmf of the noise rise [dB] in a two-services environment. The legend contains the means of the numbers of active subscribers for the various services.

In figure 2 we show the probability mass function of the noise rise for different service mixtures. We assume here a simple service mixture with only two services, a voice service with data rate 12.2kb/s and a streaming service with 384 kb/s. In particular in environments with less voice subscribers and many high data rate users we are far away from Gaussian distributions.

VI. Conclusions

We presented a model for calculating the uplink transmission powers in a "many cells-many mobiles" environment. Each mobile may use a different bearer service with different requirements. This approach is very general and the starting point for Monte-Carlo-simulations as well as for our basic model for an analytical approach.

One main subject of this analytical approach is to compute distribution functions for cell load, for received powers and for transmitting powers in the uplink. Using these functions we are able to determine coverage probabilities and call blocking rates. These information are essential for an accurate planning of 3G mobile networks based on WCDMA-technology. The basic analytic approach is implemented in T-Mobile's planning tool Pegasos® [6]. It allows a very fast analysis of all relevant UMTS planning topics like coverage of different services, blocking, cell load caused by in-cell and inter-cell interference, percentage of dropped traffic for the different services, throughput of the cells. As an example the coverage probability for an 144k service is depicted in figure 3.

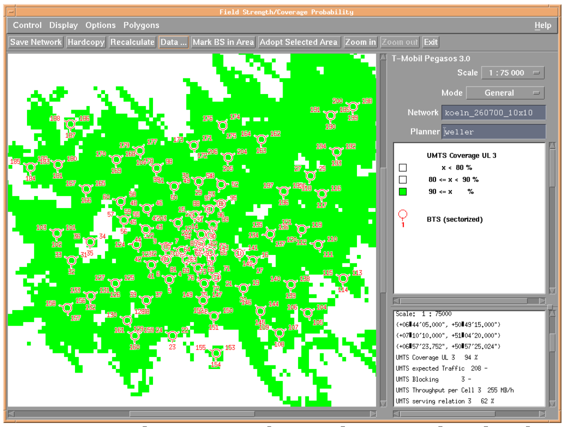

Fig. 3. A realistic network in Cologne. Colored is the area with more than 90% coverage probability for a 144kb/s UMTS service.

Although the distances between the NodeB are typically only a few hundred meters one finds some coverage holes in the planning area. The network is not very well designed for the expected very high traffic demand. As a consequence admission and load control leads to a blocking of traffic. This is shown in figure 4.

The basic analytic approach in Pegasos® is very suitable for the practical work of a UMTS rollout because all the above mentioned UMTS planning topics are calculated very fast just in one run. In Monte Carlo simulations we recommend to simulate 1000 drops of user distributions to have the analysis result stable enough. We mainly use this simulation technique for basic research, tuning the analytic model and for simulating special scenarios.

As described in section IV we take the conditioned pmf for the cell loads that at least one user of a service is active into account. In this way we avoid the problems with the concept of a "probe mobile" inherent in Monte Carlo simulations: a probe mobile scans the coverage probability at each location (bin) in the planning area. It is assumed that the probe mobile does not influence the UMTS network to avoid a new simulation. Of course this

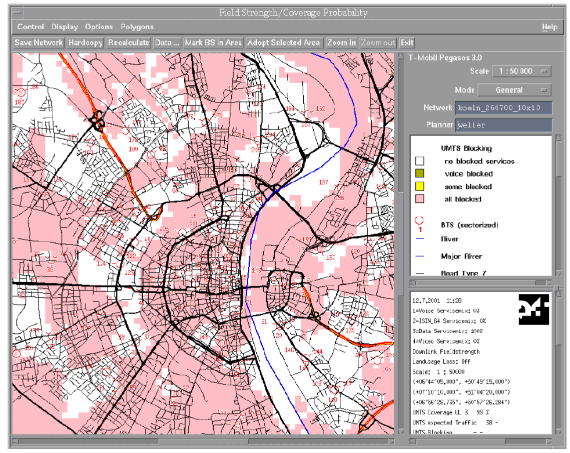

Fig. 4. The same network. The colored area shows where parts of the expected traffic of the 144kb/s UMTS service are blocked.

approximation is very good in the case of low data rates but it may lead to severe errors for high data rate services.

The next step will be to incorporate soft handover in our model, which makes the theory quite more complex. In [2] we present the effects of soft handover which we obtained from simulations. The results will allow simplifications for the analytic model.
A similar approach for the downlink will be presented in a forthcoming paper.

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