University of Würzburg
Institute of Computer Science
Research Report Series

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Andreas Mäder and Dirk Staehle

Report No. 333 May 2004

University of Würzburg,
Dept. of Computer Science,
Am Hubland, 97074 Würzburg, Germany
[maeder, staehle]@informatik.uni-wuerzburg.de

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Andreas Mäder
Department of Distributed Systems
University of Würzburg
Würzburg, Germany
Email: maeder@informatik.uni-wuerzburg.de

Dirk Staehle
Department of Distributed Systems
University of Würzburg
Würzburg, Germany
Email: staehle@informatik.uni-wuerzburg.de

***Abstract*— The *Univeral Mobile Communication System* (UMTS) operates with *Wideband Code Division Multiple Access* (WCDMA) over the air interface. The uplink capacity in WCDMA strongly depends on the interference situation in the network. The fraction of interference received at a NodeB originating from mobile stations (MS) in foreign cells is called *other–cell interference*. In the literature, this interference is often approximated by a constant factor or an independent random variable. In this paper, we propose an analytic algorithm to calculate the uplink blocking probabilities in a WCDMA network in order to answer the question how much traffic the network can carry. The algorithm considers the other–cell interference by calculating the mutual generated load between all cells in the network. Furthermore, the system model considers multiple service classes and the effects of imperfect power control.**

I. Introduction

The forthgoing build–up of the *Universal Mobile Telecommunication System* (UMTS) in Europe demands a sophisticated network planning. The primary questions which must be answered in the planning process are: Does the network cover the desired area and does the network carry the predicted traffic? The uplink capacity of a WCDMA network is limited by the amount of *Multiple Access Inteference* (MAI) received at the NodeB by all mobiles in range. Therefore the capacity is a stochastical value and is known as *soft-capacity* in contrast to the deterministic capacity in systems like GSM with *hard capacity*. The MAI at a NodeB is further differentiated into own–cell interfence, denoting interference generated by mobiles which are power–controlled by the considered NodeB, and other–cell interference generated by all other mobiles. Because the power control mechanism tries to maintain a constant received power level for all mobiles in the cell, the power received from mobiles in neighbouring cells, namely the other–cell interference, depends on the position of the mobiles relative to the considered NodeB. Furthermore, the capacities of WCDMA cells are mutual dependent, since a higher other–cell interference in cell A leads to higher transmission powers of the power controlled mobiles which in turn leads to a higher other–cell interference in neighbouring cells.

In the literature, the other–cell interference is often modelled as a relative interference factor, known as the f–factor, which is multiplied with the own–cell interference to approximate the total received interference. Some of the first papers with this approach are [1], [2] and [3]. In [4], a closed–form expression for the f–factor is developed with the precondition that the positions of the NodeBs and the mobiles are distributed according to a spatial Poisson process. Similar approaches can be found e.g. in [5] and [6]. In [7], this model is extended and used to derive a distribution for the other-cell interference. In [8], derivatives of the homogenous spatial mobile distribution are developed. The inherent problem of the f-factor approach is the assumption that the interference received from other cells is a fraction of the interference generated in the own cell. This implicates that the recieved other–cell interference is homogenous from all surrounding cells in the network.

Another approach is taken in [9], where the other–cell interference is modelled as an independent lognormal random variable. This, however, neglects the mutual dependency existing between the other– and own–cell interferences in a WCDMA network. In [10], an iterative algorithm for the calculation of the other-cell interference is proposed. The drawback of this method is the time until the iteration converges. The algorithm we propose in this paper combines both approaches, but without the need for an iterative computation.

In the remainder, we present an analytical model for calculating the blocking probabilities of a WCDMA network. The other–cell interference is calculated depending on the current system state of the considered NodeB and the load situation at the neighbouring NodeBs. The algorithm includes soft–blocking and the effects of imperfect power control. We use a modified recursive scheme based on [11] and [12], making the algorithm predestinated for the use in planning tools due to its low computational costs.

This work is organized as follows: In Sec. II, the problem of calculating the capacity of WCDMA networks is formulated more detailed. The system and interference model based on a discretized cell model are described in Sec. III. It is then applied in Sec. IV for the calculation of the blocking probabilities and the cell capacity. In Sec. V, some numerical results for selected scenarios are presented. Finally, we conclude this paper in Sec. VI.

II. Problem Formulation

In WCDMA systems like UMTS, and generally in every CDMA system, the capacity of the cellular network depends on the current interference level. Sources of interference are mobiles in the own cell[1], mobiles in the other cells and noisy signals from unknown sources. A higher received interference level at the NodeBs leads to lower signal-to-interference ratios and subsequently requires higher signal powers. This implicates that the received interferences in a WCDMA network depend on each other, since a higher own–cell interefence at an arbitrary NodeB — say NodeB x — is seen as other–cell interference in surrounding NodeBs, which requires higher signal powers in order to meet the E_b/N_0–targets leading in turn to an increased other–cell interference at x. These mutual dependencies of the interferences are illustrated in Fig. 1.

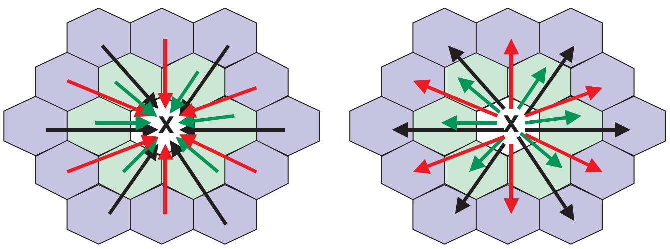

Fig. 1. Interference originating from and coming to NodeB x

So, as admission control in WCDMA networks is based on the current interference situation at the NodeB, for a more accurate modelling of the system it is important to consider these dependencies. This also implicates that the load situation in the network must be included in the calculation of the other–cell interferences.

Furthermore, for the calculation of blocking probabilities, soft–capacity which depends on interference must be considered. This implicates that in every system state an incoming call can be blocked, although the probability for this event grows with increasing cell loads. So, basically in every system state the interference situation at all NodeBs in the network must be calculated in order to obtain accurate approximations of the cell loads.

The algorithm we propose in this work calculates the blocking probabilities on the NodeBs in an WCDMA network with consideration of the effects mentioned before. Interference from other cells is calculated with inclusion of the load situation at the own and the surrounding NodeBs.

III. System and Interference Model

We define a UMTS network as a set $\mathcal{B}$ of L NodeBs with fixed positions. These NodeBs carry the traffic inside the area $\mathcal{F}$ which is subdivided into unit area elements $f \in \mathcal{F}$. The mobiles in each area element arrive according to an independent Poisson process with arrival rate λ_f. We associate an area element f to a NodeB x, $(f \in x)$ if x is the NodeB with the largest propagation gain $d_{f,x}$ from f:

$$f \in x \Leftrightarrow d_{f,x} = \max_{y\in\mathcal{B}}\{d_{f,y}\} \tag{1}$$

The propagation gain is deterministic and follows the definition given in [13]:

$$\gamma_{f,x} = -128.1 - 37.6 \cdot \log_{10}(\mathrm{dist}(f,x)) \tag{2}$$

The UMTS system further provides S different services wich are defined by the bitrate R_s and the E_b/N_0–target $\hat{\varepsilon}^*_s$. The probability that an incoming call is of service class s is given by p_s. We call the combination of the spatial traffic distribution and the service mixture a scenario. In the following, we derive the probability that an incoming call of service class s is blocked by the system. We propose a method to include the load in neighbouring cells into the calculation by considering the other–cell interference I_x^{other} at the NodeB x.

A. Interference Model

The interference at the NodeBs can be categorized after its source. We call the interference caused by mobiles which are power controlled by the NodeB x the *own–cell interference* I_x^{own}. In contrast, interference caused by all other mobiles in the network is the *other–cell interference* I_x^{other}. The background noise is denoted by N_0 and is assumed as constant at -174dBm/Hz.

The calculation of the interference and load is based on the received E_b/N_0 value for a mobile k at NodeB x write $k \in x$:

$$\hat{\varepsilon}^*_{k,x} = \frac{W}{R_k}\frac{\hat{S}^R_{k,x}}{W\hat{N}_0 + \hat{I}_x - \hat{S}^R_{k,x}\nu_k}, \tag{3}$$

where $\hat{S}^R_{k,x}$ is the received signal power from mobile k and $\hat{I}_x$ is the total received interference on NodeB x. In UMTS, the system ciprate W is 3.84Mcps. We consider imperfect power control, so the received E_b/N_0 values at the NodeBS oscillate around the E_b/N_0–target. Therefore we assume the received E_b/N_0 as normal distributed r.v. in the dB scale. This assumption has been verified by measurements e.g. in [2]. So, the receieved E_b/N_0 values are lognormal distributed with parameters

$$\mu_{\hat{\varepsilon}^*_s} = \varepsilon^*_s\frac{\ln(10)}{10} \quad \text{and} \quad \sigma_{\hat{\varepsilon}^*_s} = \sigma_{\varepsilon^*_s}\frac{\ln(10)}{10}, \tag{4}$$

where $\sigma_{\varepsilon^*_s}$ is the standard deviation in the dB scale. Note that throughout the paper, $\hat{x}$ denotes a variable not in dB scale, while x denotes a db-scaled variable.

The own–cell interference is defined as the sum of all received signal powers $\hat{S}^R_{k,x}$, which follows from Eq. (3):

$$\hat{S}^R_{k,x} = \omega_k\left(W\hat{N}_0 + \hat{I}^{own}_x + \hat{I}^{other}_x\right). \tag{5}$$

According to e.g. [9], we define the load factor of a mobile k as

$$\omega_k = \frac{\hat{\varepsilon}^*_k R_k}{W + \hat{\varepsilon}^*_k R_k}. \tag{6}$$

We omit the activity factor ν_k as a mobile contributes to the interference only if active. The own–cell interfence then

[1] Note that mobiles which are power–controlled by the considered NodeB are referred to as mobiles in the own cell

follows as the sum of all received signal powers from the k mobiles in x:

$$\hat{I}_x^{own} = \eta_x(W\hat{N}_0 + \hat{I}_x^{own} + \hat{I}_x^{other}), \tag{7}$$

with the own-cell load defined as $\eta_x = \sum_{k\in x} \nu_k \omega_k$. Solving for $\hat{I}_x^{own}$ yields

$$\hat{I}_x^{own} = \frac{\eta_x}{1-\eta_x}\left(W\hat{N}_0 + \hat{I}_x^{other}\right). \tag{8}$$

So, the total interference at NodeB x is the sum of the own–cell and other–cell interference:

$$\hat{I}_x = \hat{I}_x^{own} + \hat{I}_x^{other} = \frac{\eta_x}{1-\eta_x}\left(W\hat{N}_0 + \hat{I}_x^{other}\right) + \hat{I}_x^{other}. \tag{9}$$

Let us now investigate the relation between the received own–cell interference at NodeB x and the interference at an arbitrary NodeB y. The received signal power of a mobile k power controlled by x, $k \in x$, at NodeB y is the product of the attenuation ratio of the mobile to both NodeBs with the received power at x:

$$\hat{S}_{k,y}^R = \hat{S}_k \hat{d}_{k,y} = \hat{S}_{k,x}^R \frac{\hat{d}_{k,y}}{\hat{d}_{k,x}} = \hat{S}_{k,x}^R \hat{\Delta}_{k,y}, \tag{10}$$

where $\hat{S}_k$ is the transmission power of mobile k. We define the attenuation ratio $\hat{\Delta}_{k,y}$ of mobile k to NodeB y as the ratio of the propagation gain to NodeB y and to the controlling NodeB x. We define further

$$\omega_{k,y} = \omega_k \hat{\Delta}_{k,y} \quad \text{and} \quad \eta_{x,y} = \sum_{k\in x} \nu_k \omega_{k,y}, \tag{11}$$

where $\omega_{k,y}$ is the load factor the mobile $k \in x$ generates at NodeB y and $\eta_{x,y}$ is the own–cell load at NodeB x with respect to NodeB y. The interference received at NodeB y is then the sum of all $S_{k,y}^R$:

$$\hat{I}_{x,y} = \sum_{k\in x} \hat{S}_k^R \hat{\Delta}_{k,y} = \sum_{k\in x} \omega_{k,y}(W\hat{N}_0 + \hat{I}_x^{own} + \hat{I}_x^{other}). \tag{12}$$

Finally, replacing $\hat{I}_x^{own}$ with Eq. (8) gives us

$$\hat{I}_{x,y} = \frac{\eta_{x,y}}{1-\eta_x}\left(W\hat{N}_0 + \hat{I}_x^{other}\right). \tag{13}$$

Then, as the other–cell interference at a NodeB x comprises the interference of all surrounding NodeBs in $\mathcal{B}$, the other–cell interference is

$$\hat{I}_x^{other} = \sum_{y\in\mathcal{B}\setminus x} \hat{I}_{y,x} \tag{14}$$

B. Stochastic Fixed–Point Equation of the Other–Cell Interference

From the previous section we obtain the following stochastic fixed point equation describing the interdependencies of the interferences:

$$\hat{I}_x^{other} = \sum_{y\in\mathcal{B}\setminus x} \hat{I}_{x,y} \quad \text{and} \quad \hat{I}_{y,x} = \zeta_{y,x}(W\hat{N}_0 + \hat{I}_x^{other}), \tag{15}$$

where $\zeta_{x,y}$ is defined as

$$\zeta_{x,y} = \frac{\eta_{x,y}}{1-\eta_x}. \tag{16}$$

Note that in this equation the system loads η are random variables depending on the number of mobiles, their services and their attenuation ratios. One possibility to solve this equation is to derive the PDF of the random variables $\eta_{x,y}$ and calculate the interferences iteratively, starting with no other–cell interference at all. In the subsequent iterations, $\hat{I}_{x,y}$ is calculated under the assumption that $\zeta_{x,y}$ and the term $(W\hat{N}_0 + \hat{I}_x^{other})$ are independent. However, this method is very time consuming as the PDFs of the $\zeta_{x,y}$ have to be calculated and the iteration takes some time till convergence. Instead, as in [10], we assume that the other–cell interference is lognormal distributed which reduces the problem to the computation of the first and second moments of the other–cell interferences:

$$E[\hat{I}_x^{other}] = \sum_{y\in\mathcal{B}\setminus x} E[\hat{I}_{x,y}] \tag{17}$$

$$E[\hat{I}_{x,y}] = E[\zeta_{x,y}](W\hat{N}_0 + E[\hat{I}_x^{other}]) \tag{18}$$

The variances are given by

$$\text{Var}[\hat{I}_x^{other}] = \sum_{y\in\mathcal{B}\setminus x} \text{Var}[\hat{I}_{x,y}] \tag{19}$$

and

$$\text{Var}[\hat{I}_{x,y}] = E[(\zeta_{x,y})^2]E[(\hat{I}_x^{other})^2] - E[\zeta_{x,y}]^2 E[\hat{I}_x^{other}]^2 \tag{20}$$

Now, we compute the mean and variance of the other–cell interference by formulating these equations as matrix equations. We define the row vector

$$E\left[(\bar{I}^{other})^k\right][x] = E\left[(\hat{I}_x^{other})^k\right] \tag{21}$$

and the matrix

$$E\left[\tilde{\zeta}^k\right][x,y] = \begin{cases} E\left[(\zeta_{x,y})^k\right] & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}. \tag{22}$$

Then, Eq. (18) can be formulated as

$$E\left[\bar{I}^{other}\right] = (E\left[\bar{I}^{other}\right] + \bar{N}_0)E\left[\tilde{\zeta}\right], \tag{23}$$

where $\bar{N}_0$ is a row vector with $\bar{N}_0[x] = W\hat{N}_0$ as entries. The row vector with the mean other–cell interferences is then computed by matrix inversion:

$$E\left[\bar{I}^{other}\right] = \bar{N}_0 E\left[\tilde{\zeta}\right]\left(\tilde{\mathcal{I}} - E\left[\tilde{\zeta}\right]\right)^{-1} \tag{24}$$

The second moment of the other–cell interference at NodeB x is given by

$$\begin{aligned} E\left[\left(\hat{I}_x^{other}\right)^2\right] &= E\left[\hat{I}_x^{other}\right]^2 - \sum_{y\in\mathcal{B}\setminus x} E\left[\hat{I}_y^{other}\right]^2 E[\zeta_{x,y}]^2 \\ &+ \sum_{x\in\mathcal{B}\setminus x} \left((W\hat{N}_0)^2 + 2W\hat{N}_0 E\left[\left(\hat{I}_y^{other}\right)\right]\right) \text{Var}[\zeta_{x,y}] \\ &+ \sum_{y\in\mathcal{B}\setminus x} E\left[\left(\hat{I}_y^{other}\right)^2\right] E\left[(\zeta_{x,y})^2\right]. \end{aligned} \tag{25}$$

Furthermore, we introduce the variable H_y defined as

$$H_y = E\left[\hat{I}_x^{other}\right]^2 - \sum_{y\in\mathcal{B}\setminus x} E\left[\hat{I}_y^{other}\right]^2 E[\zeta_{x,y}]^2 + \sum_{y\in\mathcal{B}\setminus x}\left((W\hat{N}_0)^2 + 2W\hat{N}_0 E\left[\left(\hat{I}_y^{other}\right)\right]\right)\mathrm{Var}[\zeta_{x,y}] \tag{26}$$

The row vector $\bar{H}$ consists of entries H_y for all $y \in \mathcal{B}$. Then, the matrix equation for the second moment of the other–cell interference is formulated:

$$E\left[\left(\bar{I}^{other}\right)^2\right] = \bar{H}\left(\tilde{\mathcal{I}} - E\left[\tilde{\zeta}^2\right]\right)^{-1}. \tag{27}$$

In Fig. 2, the mean other–cell interferences at all NodeBs in a hexagonal cell layout with two tiers are shown. The NodeBs are ordered from the center to the edge of the network, so the first column indicates the NodeB in center of the network. The mean offered load η_{norm} is normalized to the NodeB with the highest offered load. For lower offered loads, the analytic and the simulated results match very well. For a cell load $\eta_{norm} = 0.6$ the values begin to diverge, but are still quite accurate.

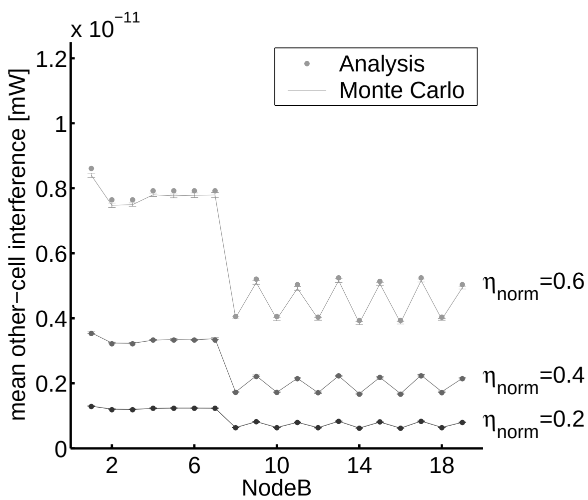

Fig. 2. Mean other–cell interferences in a hexagonal cell layout

The calculation of the other–cell interferences requires the moments of the r.v. ζ_x and $\zeta_{x,y}$. We describe an approximation method for these moments in the next section.

C. Moments of ζ_x and $\zeta_{x,y}$

The derivation of the moments of $\zeta_{x,y}$ requires the moments of the cell load η_x. We obtain these recursively from the user Markov process which is described in detail in the next section. Since η_x is the sum of lognormal distributed random variables ω_k, η_x is lognormal distributed too and we denote its PDF as $\phi_{\eta_x}(t)$. Then, the PDF of ζ_x follows directly as

$$\phi_{\zeta_x}(t) = \phi_{\eta_x}\left(\frac{t}{1+t}\right). \tag{28}$$

We compute the first and second moments by numerical integration. However, due to the fact that ζ_x goes to infinity for $\eta \to 1$, we calculate the moments of ζ_x under the condition that the system load η_x is less than $1-\epsilon$. This restriction is also justified by the real world behavior of WCDMA systems where loads near the pole capacity lead to infinite interferences.

The cell load inflicted to NodeB y, $\eta_{x,y}$, includes the attenuation ratios $\hat{\Delta}_{k,y}$ of the mobiles $k \in x$, so the computation of the moments becomes more complicated. However, under the assumption that the load factors ω_k and the attenuation ratios are independent, the service and traffic component is separated from the spatial component and we write:

$$E[\zeta_{x,y}] = E\left[\frac{\sum_{k\in x}\omega_k\hat{\Delta}_k}{1-\sum_{k\in x}\omega_k}\right] = E[\zeta_x]E[\hat{\Delta}_{x,y}] \tag{29}$$

The assumption of independency is valid if all services follow the same spatial distribution and we introduce the random variable $\hat{\Delta}_{x,y} = \hat{\Delta}_{k,y}$ for all $k \in x$. The same principle — separation of the service and the spatial component — is used for the calculation of the second moment of $\zeta_{x,y}$. It can be shown that the second moment is

$$E[(\zeta_{x,y})^2] = E[(\zeta_x)^2]E[\hat{\Delta}_{x,y}]^2 + E[\zeta_x^2]\mathrm{Var}[\hat{\Delta}_{x,y}]. \tag{30}$$

Note that $(\zeta_x)^2$ is different from the r.v. ζ_x^2 which is defined as

$$\zeta_x^2 = \frac{\eta_x^2}{1-\eta_x} \quad \text{and} \quad \eta_x^2 = \sum_{k\in x}\omega_k^2 \tag{31}$$

The moments of the spatial component $\Delta_{x,y}$ follow from the conditioned sum of all subelements f in the considered area $\mathcal{F}$. Neglecting soft handover, we say that an area element f belongs to the NodeB with the largest propagation gain, say x, with the probability $p(f \in x)$:

$$p(f \in x) = P\left(\hat{d}_{x,f} = max_y\left\{\hat{d}_{y,f}\right\}\right) \tag{32}$$

The traffic intensity of service s ate the NodeB x then becomes

$$a_{x,s} = p_s\sum_{f\in\mathcal{F}} a_f p(f \in \mathcal{F}) \tag{33}$$

and the moments of $\hat{\Delta}_{x,y}$ are

$$E[\hat{\Delta}_{x,y}^k] = \sum_{f\in\mathcal{F}}\frac{a_f p(f\in\mathcal{F})}{\sum_{s=1}^S a_{x,s}} E\left[\left(\frac{\hat{d}_{y,f}}{\hat{d}_{x,f}}\right)^k \middle| f \in \mathcal{F}\right]. \tag{34}$$

We now have the means to calculate the total received interference a the NodeBs depending on the current system load, that is on the number of mobiles in the cell. Together with the calculation of the soft-blocking probabilities in each system state, the total blocking probabilities of the system are obtained in the next section.

IV. WCDMA Cell Capacity

The admission control (AC) in UMTS is based on the received interference. Since the interference is a stochastical value, blocking of new calls can occur in virtual every system state. The term *local soft blocking probability* refers to this characteristic of the system.

A. Local Soft Blocking Probabilities

An incoming call is blocked if the estimated cell load η is higher than a predefined maximum cell load $\eta_{max} < 1$. We define the cell load as

$$\eta = \frac{\hat{I}_x^{own} + \hat{I}_x^{other}}{\hat{I}_x^{own} + \hat{I}_x^{other} + \hat{N}_0} < \eta_{max} \quad (35)$$

deriving from the definition of the *noise rise*, see e.g. [6]. This can be reformulated to

$$\eta_x + \omega_s + \Gamma(\eta_x) < \eta_{max}, \quad (36)$$

where the r.v. $\Gamma(\eta_x)$ denotes the cell load induced by the other–cell interference depending on the own–cell load η_x. It is defined as

$$\Gamma(\eta_x) = \hat{I}_x^{other}(\eta_x)\frac{1-\eta_{max}}{\hat{N}_0}. \quad (37)$$

So, the probability $\beta_s(\eta_)$ that for a given η_x the call of a mobile with service class s is blocked at the NodeB x is

$$\beta_s(\eta_x) = 1 - P(\eta_x + \omega_s + \Gamma(\eta_x) < \eta_{max}). \quad (38)$$

Note that in this case the activity factor ν is omitted since we assume that an incoming connection is always active. All r.v. in Eq. (38) are assumed lognormal, so

$$\beta_s(\eta_x) = 1 - \mathrm{LN}_{\mu_x,\sigma_x}(\eta_{max}), \quad (39)$$

where μ_x is derived from the first moment of the total cell load as

$$\mu_x = \ln(E[\eta_x] + E[\omega_s] + E[\Gamma(\eta_x)]) - \tfrac{1}{2}\sigma \quad (40)$$

and the shape parameter σ from the variance as

$$\sigma = \sqrt{\ln\left(\frac{\mathrm{Var}[\eta_x] + \mathrm{Var}[\omega_s] + \mathrm{Var}[\Gamma(\eta_x)]}{E[\eta_x] + E[\omega_s] + E[\Gamma(\eta_x)]}\right)^2 + 1}. \quad (41)$$

Note that η_x and $\Gamma(\eta_x)$ are correlated due to the effects of the imperfect power control. However, the spatial effects on the r.v. $\zeta_{x,y}$ do outweigth this effect by far, so we assume independence.

B. Analytic Calculation of the Total Blocking Probabilities

The cell loads η_x, the other–cell interference $\hat{I}_x^{other}$ as well as the other–cell load $\Gamma(\eta_x)$ and the soft blocking probabilities depend on the system state, that is on the number of mobiles power–controlled by NodeB x. We obtain these values by defining a modified Markov process where the transition rates are conditioned with the local soft blocking probabilities, see e.g. [14]. The state probabilites are calulated recursively. In every recursion step, i.e. for every system state, the other–cell interference is computed by matrix inversion as in Eq. (24). To consider the loads at the surrounding NodeBs $y \neq x$, the first and second moments of $\zeta_{x,y}$ are calculated in beforehand without considering soft–blocking. These are then used as input for the calculation of the other–cell interference at x in the recursion algorithm.

For the recursive calculation scheme of the state probabilities following [11] and [12], states with similar resource occupations, that is with similar values of η_x, are combined to one state. This folds the S-dimensional state space into one dimension, with transitions for the different resource requirements of the service classes.

In order to combine similar states, a common resource must be defined. In this case it is reasonable to choose the load factor η_x as resource, with the condition that $\eta_x < \eta_{max}$. The maximal load η_{max} is implicitly given by the condition that it must hold that $\eta_x < 1$ for the feasibility of the power control equation. So, we define a basic resource unit g and map the service load factors ω_s with the activity factor ν_s to resource requirements ψ_s which are multiples of g:

$$\psi_s = \left(\lfloor \tfrac{\nu_s \omega_s}{g} + \tfrac{1}{2} \rfloor\right) g \quad (42)$$

Note that the maximum cell load η_{max} should be a integer multiple of g.

The recursion algorithm defined in [11] must be modified in order to include the local soft blocking probabilities. This leads to an approximation error since the recursion formula assumes that transitions in the same dimension have equal transition rates, which does not hold here because of the soft blocking. The modified recursion formula then becomes

$$\tilde{p}(\eta^*) = \tfrac{1}{\eta^*}\sum_{s=1}^{S}(1-\beta_s(\eta^* - \psi_s))\tilde{p}(\eta^* - \psi_s)a_s\psi_s, \quad (43)$$

where η^* is the current system state and is a integer multiple of g. The state probability follows by normalizing $\tilde{p}$:

$$p(\eta^*) = \frac{\tilde{p}(\eta^*)}{\sum\limits_{jg \leq \eta_{\max}} \tilde{p}(jg)}, \quad j \in \mathbb{N}_0 \quad (44)$$

The mean and the variance of the cell load η_x are also obtained recursivley. We initialize both to zero for $\eta^* = 0$ and write

$$E[\eta_x(\eta^*)] = \sum_{s=1}^{S} P_s(\eta^*)(E[\eta_x(\eta^* - \phi_s) + \nu_s E[\omega_s]) \quad (45)$$

$$\begin{aligned} E[\eta_x(\eta^*)^2] = \sum_{s=1}^{S} & P_s(\eta^*)(E[\eta_x(\eta^* - \phi_s)^2] \\ & + 2\nu_s E[\eta_x(\eta^* - \phi_s)]E[\omega_s] + \nu_s E[\omega_s^2]) \end{aligned} \quad (46)$$

for $0 < \eta^* < \eta_{max}$. The first and second moments of ζ_x, $\zeta_{x,y}$ and $\zeta_{x,y}^2$ follow according to Eq. (29), (30) and (31). Note that we assume that the activities of mobiles with same service classes are equal, hence $\nu_s = \nu_k$.

The probability $P_s(\eta^*)$ denotes the conditional probability that the current system state η^* has been reached from the predecessing state $\eta^* - \psi_s$ by an incoming connection of service class s. This probability is given by

$$P_s(\eta^*) = \frac{(1-\beta_s(\eta^*))\tilde{p}(\eta^*)a_s\psi_s}{\sum\limits_{t=1}^{S}(1-\beta_t(\eta^*))\tilde{p}(\eta^*)a_t\psi_t}. \quad (47)$$

Finally, the total blocking probabilities can be calculated as the sum over all state probabilities multiplied with the corresponding local soft blocking probabilities:

$$P_{\text{block}}(s) = \sum_{jg < \eta_{\max}} \beta_s(jg)p(jg), \quad j \in \mathbb{N}_0 \quad (48)$$

V. Numerical Results

In the following, numerical results will be shown for an example UMTS network with a hexagonal cell layout with two tiers as shown in Fig. 1. The results are validated with an event-driven simulation designed to verify the assumption we made. The confidence level for all results is 0.975.

We consider two scenarios, one with two and one with three service classes with 12.2kbps, 64kpbs and 144kbps. The E_b/N_0–targets are 5.5dB, 4dB and 3dB, respectively. The standard deviations of the E_b/N_0–targets are 1.2dB for both services and scenarios. The distance between the NodeBs is 1.2km and the area element size is 50m. The maximum allowed cell load η_{max} is 0.5. For the first scenario, the service probabilites p_s are 0.6 for the first and 0.4 for the second service. For the second scenario, the service probabilities are 0.5, 0.3 and 0.2. The mean offered load is normalized to the NodeB with the maximum offered load.

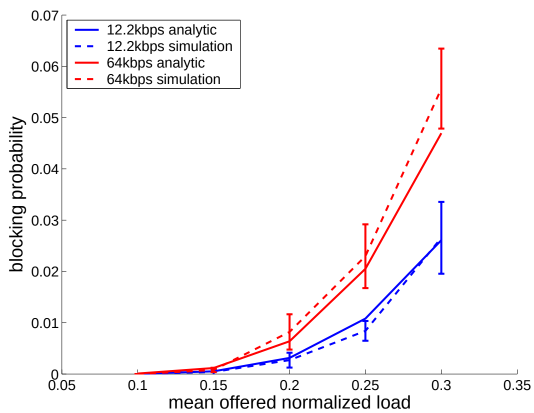

Fig. 3. Blocking probabilities for two service classes

In Fig. 3, the analytic results for the service class with the lower bitrate match very well with the simulated results, while the analytic results for the higher service class underestimate the simulation. In Fig. 4, the case is similar: The analytic results for the first two service classes with 12.2kbps and 64kbps match the simulation quite accurate, while the highest service class is underestimated by the analytic algorithm.

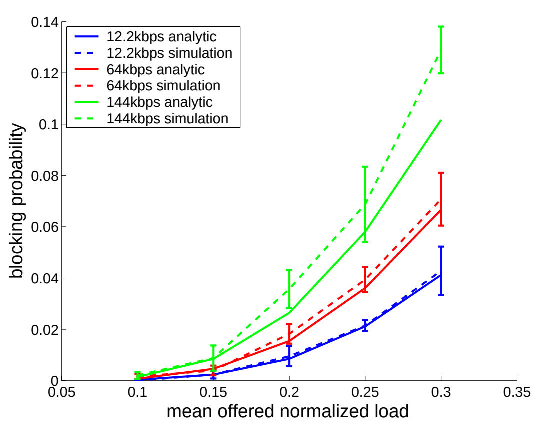

Fig. 4. Blocking probabilities for three service classes

VI. Conclusion

The goal of this paper was to develop an analytical model for the calculation of blocking probabilities in a UMTS network. Special attention was turned to the other–cell interference, whose contribution to the overall cell load must be considered for an accurate modelling of WCDMA systems like UMTS. For this reason, we proposed an analytic algorithm which computes the other–cell interference at the considered NodeB for every system state. Together with the own–cell interference this enabled us to calculate the soft blocking probabilities reflecting the admission control mechanism in WCDMA. The approximation of the other–cell interference showed very good results for lower cell loads and sufficient results for high cell loads. The blocking probabilities are underestimated, but nevertheless give a good impression for blocking targets relevant to network operators. This and the low computational costs of the algorithm due to its recursive scheme makes it a valuable tool for the planning of WCDMA networks.

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