

# Verified Parameter Identification for Dynamic Systems with Non-Smooth Right-Hand Sides

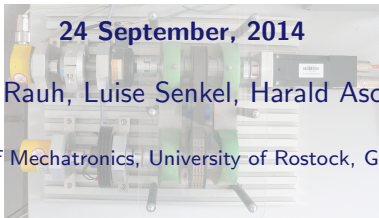
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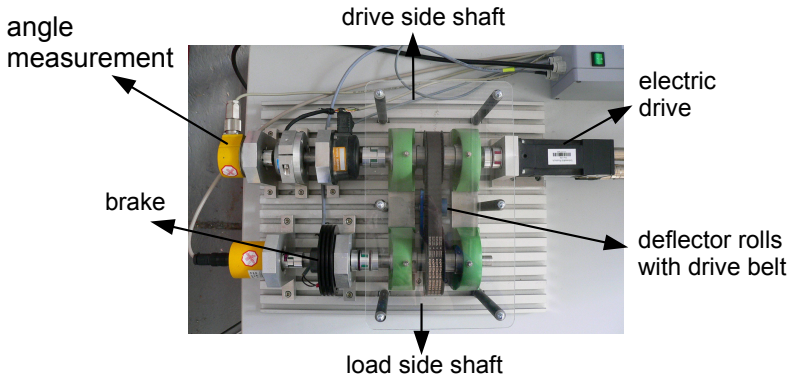


# Contents

- Motivation: Verified parameter identification for a drive train test rig
- System significantly influenced by static/ sliding friction
- Necessity for handling transitions between different discrete model states (non-smooth right-hand side of an ODE)
- Verified integration approach for corresponding initial value problems
- Identification procedures
- Experimental results and comparison with an interval-based sliding mode observer
- Conclusions and outlook on future work

# Verified Parameter Identification for a Drive Train Test Rig

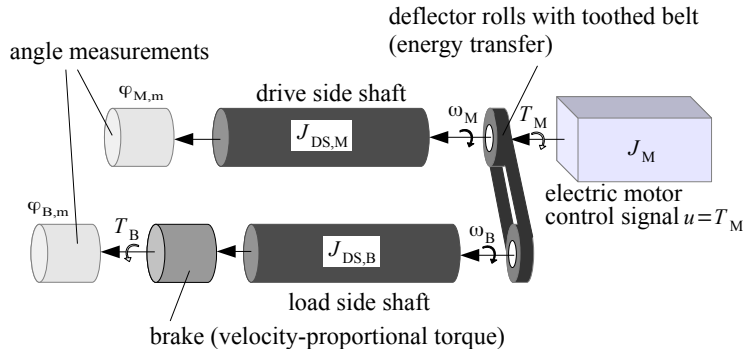
## Experimental setup



- Identification of static friction, sliding friction
- Identification of mass moment of inertia

# Verified Parameter Identification for a Drive Train Test Rig

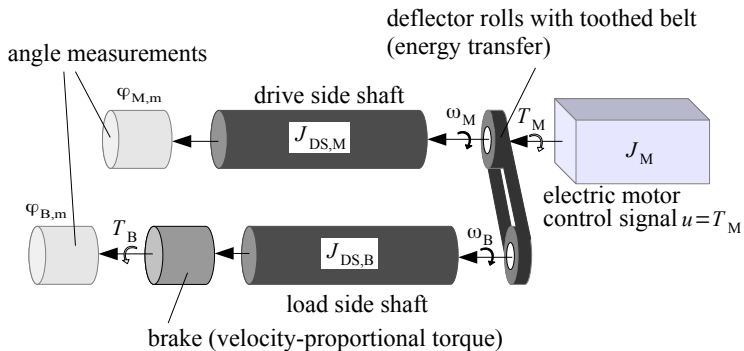
## Experimental setup



- Measurements: Angle(s) and angular velocities
- Actuation of brake: Non-modeled disturbance

# Verified Parameter Identification for a Drive Train Test Rig

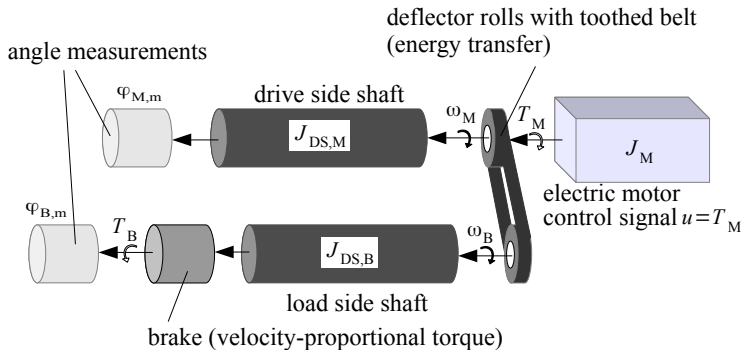
System model: Sliding friction for  $\dot{\varphi}_M = \omega_M = x_2(t) \neq 0$



$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ \alpha \cdot x_2(t) + \beta \cdot (u(t) - T_F(t)) \end{bmatrix}, \quad T_F(t) = T_{F,s} \cdot \text{sign}(x_2(t))$$

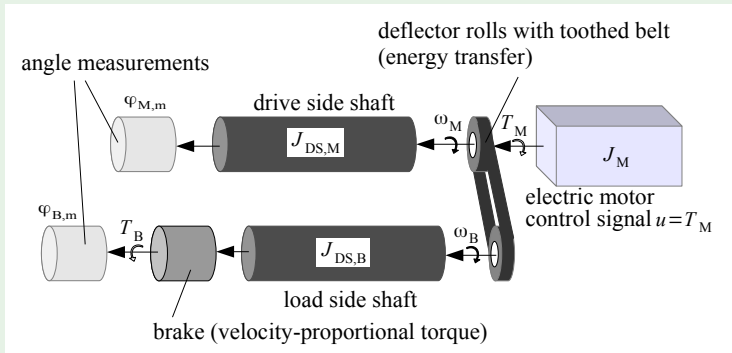
# Verified Parameter Identification for a Drive Train Test Rig

System model: Static friction for  $x_2(t) = 0$  and  $|u(t)| \leq T_{F,s}$



$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

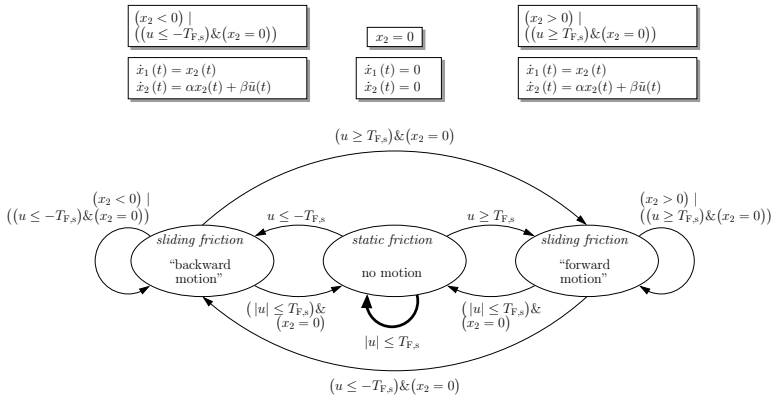
# Verified Parameter Identification for a Drive Train Test Rig



- $\alpha$  – (velocity-proportional friction)/(mass moment of inertia)
- $\beta$  –  $1/(\text{mass moment of inertia})$
- $T_{F,s}$  – static friction, possibly varying after standstill

# System Model with Non-Smooth Right-Hand Side (1)

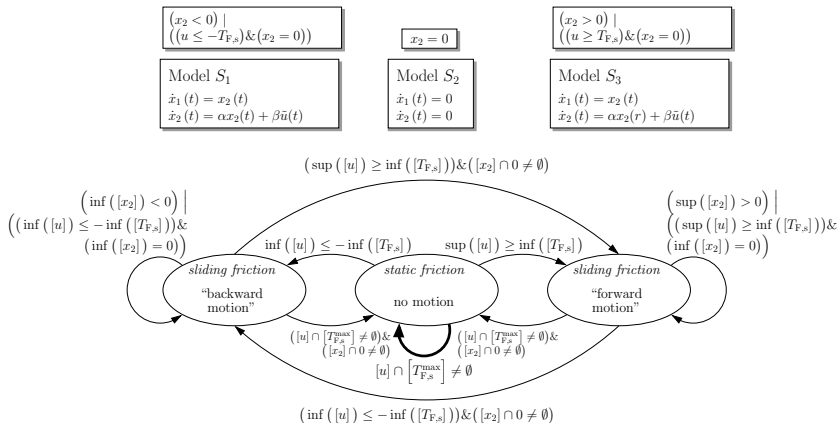
Nominal system model,  $\tilde{u}(t) := u(t) - T_F(t)$





# System Model with Non-Smooth Right-Hand Side (2)

Uncertain model,  $\tilde{u}(t) := u(t) - T_F(t)$ ,  $[T_F^{\max}] := [-\bar{T}_{F,s} ; \bar{T}_{F,s}]$



## Taylor Series-Based Enclosure Method (1)

- Discretization of considered time horizon
- Taylor series expansion of solution of the IVP with respect to time according to

$$\mathbf{x}(t_{k+1}) = \mathbf{x}(t_k) + \sum_{i=1}^{\nu} \frac{h^i}{i!} \mathbf{f}^{(i-1)}(\mathbf{x}(t_k), \mathbf{p}, \mathbf{u}(t_k), t_k) \\ + \mathbf{e}(\mathbf{x}(\xi), \mathbf{p}, \mathbf{u}(\xi), \xi), \quad \dot{\mathbf{x}}(t_k) := \mathbf{f}(\mathbf{x}(t_k), \mathbf{p}, \mathbf{u}(t_k), t_k)$$

with the integration step-size  $h$ , i.e.,  $t_k = kh$ ,  $t_{k+1} = (k+1)h$ , and  $t_k \leq \xi \leq t_{k+1}$

### Note

- System parameters  $\mathbf{p} \in [\mathbf{p}]$  are piecewise constant
- Changes of control signals  $\mathbf{u}(t_k)$  only occur at the points  $t = t_k$

## Taylor Series-Based Enclosure Method (2)

- Recursive computation of the total derivatives  $\mathbf{f}^{(i-1)}$  (resp. Taylor series coefficients) in terms of the smooth right-hand side of the ODE with  $\dot{\mathbf{p}} = \mathbf{0}$  and  $\dot{\mathbf{u}}(t) = \mathbf{0}$ ,  $t \in (t_k ; t_{k+1})$
- Calculation of guaranteed bounds of the discretization error

$$\mathbf{e}(\mathbf{x}(\xi), \mathbf{p}, \mathbf{u}(\xi), \xi) \subseteq [\mathbf{e}_k] := \frac{h^{\nu+1}}{(\nu+1)!} \mathbf{f}^{(\nu)}([\mathbf{B}_{x,k}], [\mathbf{p}], \mathbf{u}([\tau_k]), [\tau_k])$$

- Prerequisites: Differentiability of  $\mathbf{f} \in C^\nu$
- Bounding box  $[\mathbf{B}_{x,k}]$ , parameter and control enclosures  $[\mathbf{p}]$  and  $\mathbf{u}([\tau_k])$  for the time interval  $[\tau_k] := [t_k ; t_{k+1}]$  have to be available

$\Rightarrow$  Use of the Picard iteration to determine  $[\mathbf{B}_{x,k}]$

## Extension to Systems with Non-Smooth Right-Hand Sides

**Step 1** Calculation of bounding box  $[\mathbf{B}_{a,k}]$  for the time interval  $[\tau_k]$  for the union of all system models which are *active* at  $t = t_k$  with a continuously differentiable function  $\mathbf{f}_a$  enclosing the right-hand sides of **all active** models

**Step 2** Check for additionally activated models

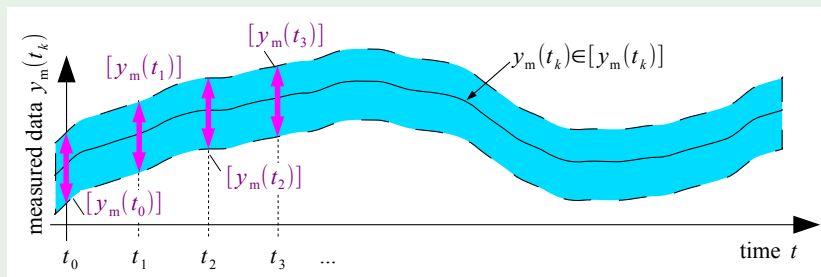
- Repeat **Step 1** if additional models are activated  
 $\implies$  modification of  $\mathbf{f}_a$  by consideration of additionally activated models
- Otherwise, continue with **Step 3**

**Step 3** Interval evaluation of series expansion for  $\mathbf{f}(\cdot) = \mathbf{f}_a(\cdot)$   
 Subsequently:  $\nu \equiv 1$

**Step 4** Deactivation of system models which can no longer be active at  $t = t_{k+1}$

# Verified Methods for Parameter Identification

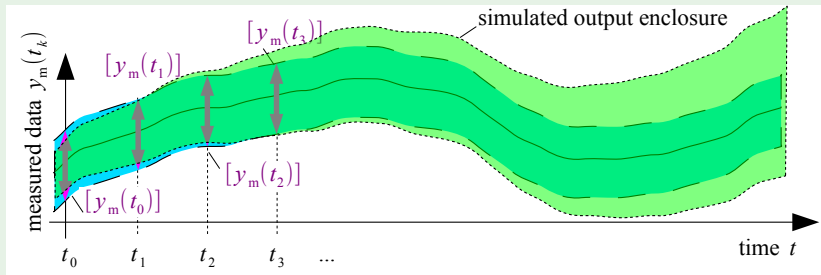
Offline procedure: Multiple simulations over complete horizon of gathered measured data



- Measured data are available at discrete points of time
- Worst-case bounds for measurement tolerances
- Information about uncertain initial states and bounds on uncertain parameters

# Verified Methods for Parameter Identification

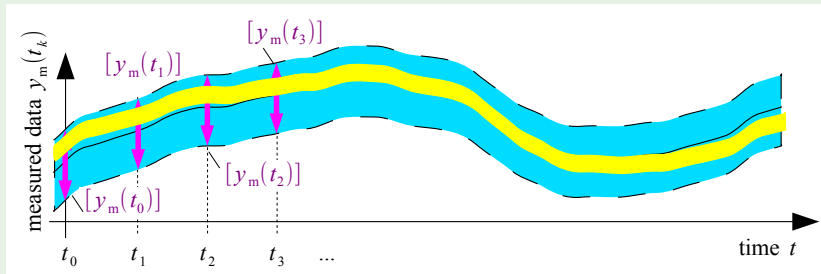
Offline procedure: Multiple simulations over complete horizon of gathered measured data



- Prerequisite: Correctness of model structure
- Initial state/ parameter intervals are subdivided for candidates, for which no decision about admissibility can be made
- Intersection of directly measured and simulated state intervals possible

# Verified Methods for Parameter Identification

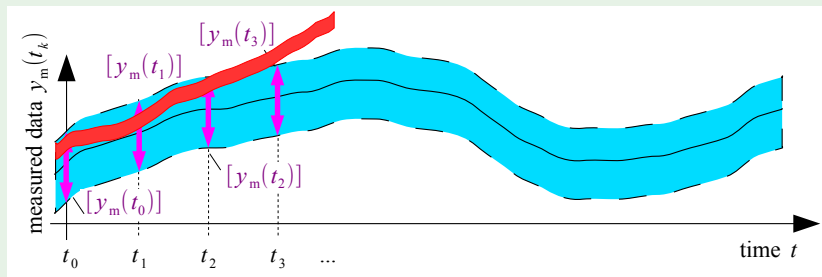
Offline procedure: Multiple simulations over complete horizon of gathered measured data



- Search for guaranteed admissible initial state/ parameter intervals
- Subdivision until undecided region is sufficiently small
- Needs to be fulfilled for each available sensor if  $\dim(\mathbf{y}_m) > 1$

## Verified Methods for Parameter Identification

Offline procedure: Multiple simulations over complete horizon of gathered measured data

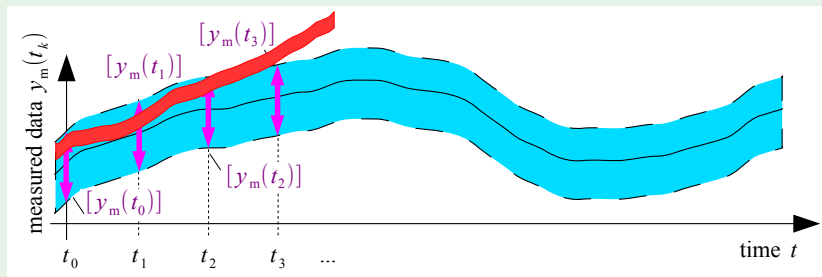


- Exclusion of inadmissible intervals (for at least one of the sensors)
- Drawback: Conservativeness for systems with non-smooth right-hand sides (large number of subintervals)



## Verified Methods for Parameter Identification

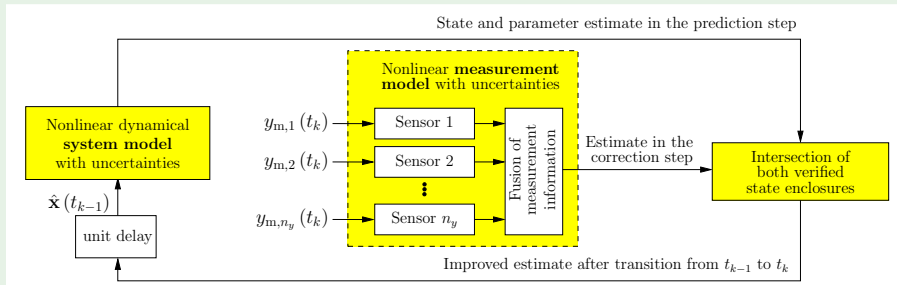
Offline procedure: Multiple simulations over complete horizon of gathered measured data



- Exclusion of inadmissible intervals (for at least one of the sensors)
- Drawback: Conservativeness for systems with non-smooth right-hand sides (large number of subintervals)
- **Parameter reset (after standstill) cannot be handled efficiently**

# Verified Methods for Parameter Identification

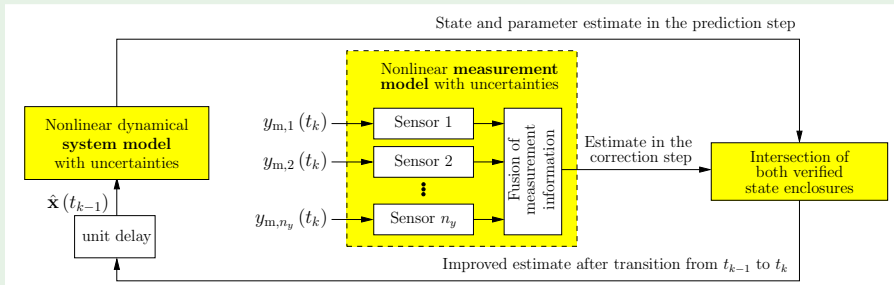
## Online procedure: Observer-based approach



- Verified integration of state equations between two subsequent measurement points  $\implies$  Structure close to Luenberger observer/ (Extended) Kalman Filter
- Exclusion of inadmissible intervals

# Verified Methods for Parameter Identification

## Online procedure: Observer-based approach



- Verified integration of state equations between two subsequent measurement points  $\implies$  Structure close to Luenberger observer/ (Extended) Kalman Filter
- Exclusion of inadmissible intervals
- **Parameter reset easily possible at specific points of time**

# Observer-Based Verified Parameter Identification (1)

- 1 Description of the state enclosure by a list of  $L$  interval boxes

$$\left[ \mathbf{z}^{<l>}(t_k) \right] := \begin{bmatrix} \left[ \mathbf{x}^{<l>}(t_k) \right] \\ \left[ \mathbf{p}^{<l>}(t_k) \right] \end{bmatrix}, \quad l = 1, \dots, L$$

- 2 Perform  $M$  subdivisions of intervals, if at least one interval  $l$  is characterized by

$$\prod_{j=1}^{n+n_p} \text{diam} \left\{ \left[ z_j^{<l>}(t_k) \right] \right\} \neq 0$$

$\implies$  New interval list of length  $L + M - 1$

- 3 Verified integration of the IVP until the next measurement point  $t_{k+1}$   
 $\implies \left[ \mathbf{z}^{<l>}(t_{k+1}) \right]$

## Observer-Based Verified Parameter Identification (2)

- ④ Intersection of all interval boxes with measured data  
 $z_1(t_{k+1}) \in [y_m(t_{k+1})]$  (assumption first state is directly measurable)

$$[\tilde{z}_1^{<l>}(t_{k+1})] := [z_1^{<l>}(t_{k+1})] \cap [y_m(t_{k+1})]$$

- ⑤ Replace  $[z_1^{<l>}(t_{k+1})]$  by  $[\tilde{z}_1^{<l>}(t_{k+1})]$  for all  $l = 1, \dots, L + M - 1$
- ⑥ Delete all subintervals with  $[\tilde{z}_1^{<l>}(t_{k+1})] = \emptyset$  from the interval list
- ⑦ Replace static friction subintervals with initial range if standstill is detected for a minimum time span (by binary signal from velocity sensor)
- ⑧ Reduce the number of subintervals by a convex hull with sufficiently small overestimation: New list length  $L := L^*$

Note: Interval replacement (step 7) and reduction of interval number (step 8) can be employed interchangeably

## Bisectioning Strategy in Step 2

### Selection of the candidates to be subdivided

$$l^* = \arg \max_{l=1, \dots, L'} \prod_{j=1}^{n+n_p} \text{diam} \left\{ \left[ z_j^{<l>}(t_k) \right] \right\}, \quad L' \geq L$$

### Reduce ambiguities between static and sliding friction

- Split static friction interval if

$$[u(t_k)] \cap \text{hull} \left\{ - \left[ T_{F,s}^{<l^*>} \right], \left[ T_{F,s}^{<l^*>} \right] \right\} \neq \emptyset$$

## Bisectioning Strategy in Step 2

### Selection of the candidates to be subdivided

$$l^* = \arg \max_{l=1,\dots,L'} \prod_{j=1}^{n+n_p} \text{diam} \left\{ \left[ z_j^{<l>}(t_k) \right] \right\}, \quad L' \geq L$$

### Reduce ambiguities between static and sliding friction

- Select splitting point  $\bar{u}(t_k) + \epsilon$ ,  $\epsilon > 0$  for  $[u(t_k)] > 0$  with  $\underline{T}_{F,s}^{<l^*>} < \underline{u}(t_k)$  and  $\bar{T}_{F,s}^{<l^*>} > \bar{u}(t_k)$
- Select splitting point  $\underline{u}(t_k) - \epsilon$ ,  $\epsilon > 0$  for  $[u(t_k)] < 0$  with  $-\bar{T}_{F,s}^{<l^*>} < \underline{u}(t_k)$  and  $-\underline{T}_{F,s}^{<l^*>} > \bar{u}(t_k)$
- Else: Splitting of  $\left[ T_{F,s}^{<l^*>} \right]$  at its midpoint

## Bisectioning Strategy in Step 2

### Avoid unnecessarily conservative interval bounds

- Split angular velocity interval  $[x_2^{<l^*>}]$  for  
 $\text{diam} \{ [x_2^{<l^*>}] \} \geq \text{diam} \{ [\beta^{<l^*>}] \}$
- Split interval  $[\beta^{<l^*>}]$  for

$$\left( [\alpha^{<l^*>}] \cdot [x_2^{<l^*>}] \right) \cap \left( [\beta^{<l^*>}] \cdot ([u(t_k)] - [T_{F,s}^{<l^*>}]) \right) \neq \emptyset$$

- Else: Split interval  $[\alpha^{<l^*>}]$
- Optional: Trisectioning of  $[x_2^{<l^*>}]$  if static and sliding friction are possible simultaneously



## Replacement of Static Friction Interval in Step 7

Repeat for each subinterval  $l = 1, \dots, L + M - 1$

- Initial range for  $T_{F,s}$ :  $\left[ T_{F,s}^{\text{ini}} \right]$
- Create subintervals

$$[T_a] := \left[ \underline{T}_{F,s}^{\text{ini}} ; \underline{T}_{F,s}^{<l>} \right]$$

$$[T_b] := \left[ \underline{T}_{F,s}^{<l>} ; \overline{T}_{F,s}^{<l>} \right]$$

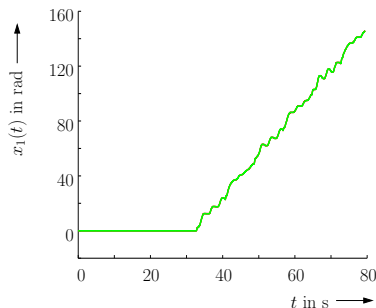
$$[T_c] := \left[ \overline{T}_{F,s}^{<l>} ; \overline{T}_{F,s}^{\text{ini}} \right]$$

- Create a list of up to  $3L$  subintervals, where  $\left[ T_{F,s}^{<l>} \right]$  is replaced by each of the intervals  $[T_a]$ ,  $[T_b]$ ,  $[T_c]$  with non-zero diameter
- Subsequent merging (not necessarily after each time step) avoids combining intervals with different active model states  $S_i$ ,  $i = 1, 2, 3$

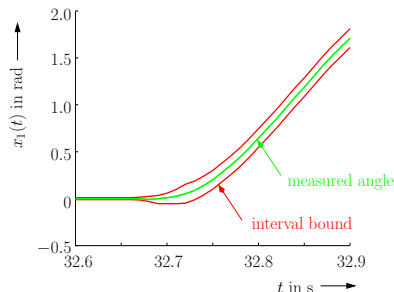
# Experimental Results for Verified Parameter Identification

- Measurement uncertainty:  $[-0.1 ; 0.1]$  rad
- Piecewise const. control: exactly known, interval hull over exp. data
- Sampling time: 10 ms

Estimate of the angle  $x_1 = \varphi_M$



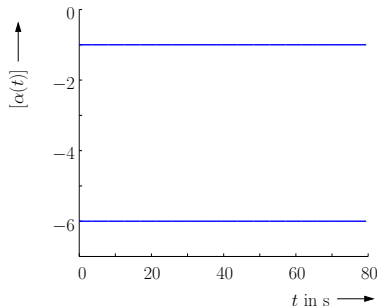
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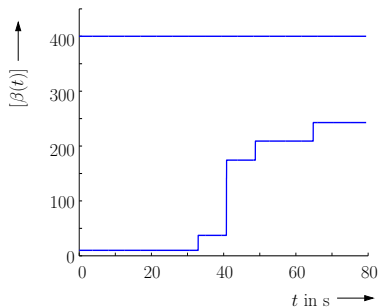
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Parameter estimate  $\alpha$



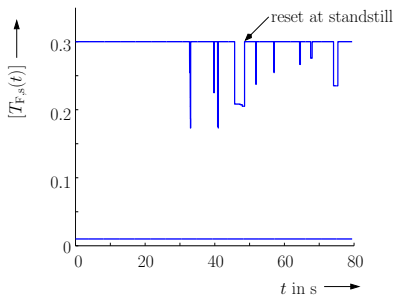
Parameter estimate  $\beta$



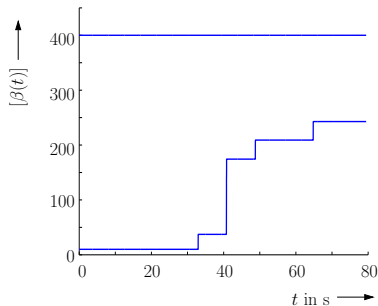
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Parameter estimate  $T_{F,s}$



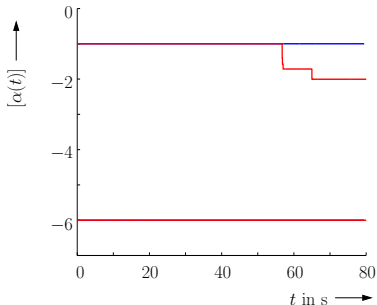
Parameter estimate  $\beta$



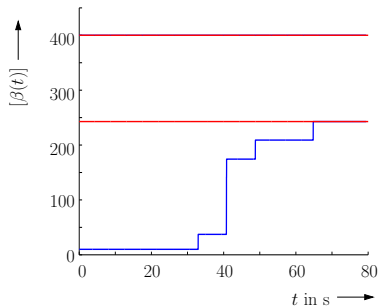
# Experimental Results for Verified Parameter Identification

- Measurement uncertainty:  $[-0.1 ; 0.1]$  rad
- Piecewise const. control: exactly known, interval hull over exp. data
- Sampling time: 10 ms, **reinitialization after 80 s with last estimate**

Parameter estimate  $\alpha$



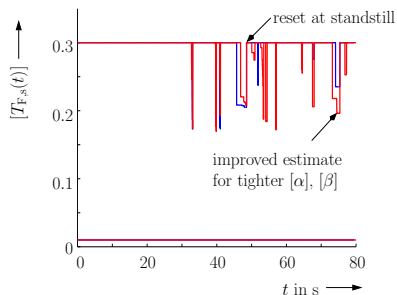
Parameter estimate  $\beta$



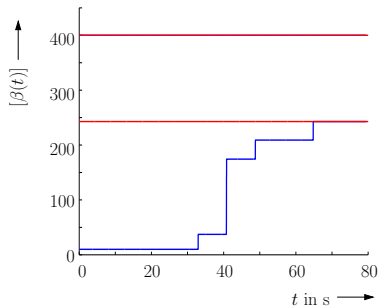
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Parameter estimate  $T_{F,s}$



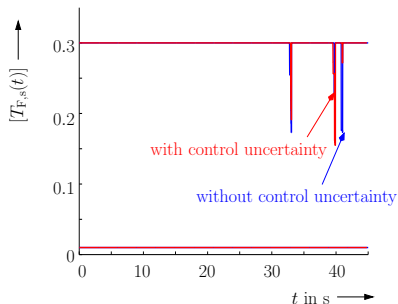
Parameter estimate  $\beta$



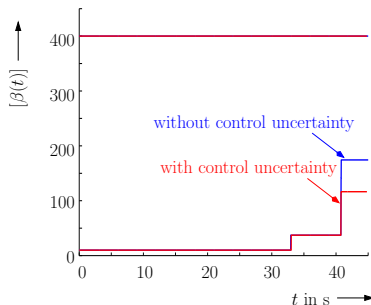
# Experimental Results for Verified Parameter Identification

- Measurement uncertainty:  $[-0.1 ; 0.1]$  rad
- Piecewise const. control: **uncertainty: 10% of the experimental data**
- Sampling time: 10 ms

Parameter estimate  $T_{F,s}$



Parameter estimate  $\beta$



## Conclusions and Outlook on Future Work

- Verified integration of ODEs with non-smooth right-hand side
- Observer-based identification of parameter intervals
- Consistency with dynamic system model and (uncertain) measured data
- Efficiency of subdivision heuristics
- Validation of alternative estimation procedures (e.g. sliding mode)  
⇒ Use of interval bounds for system parameters to describe the stability domains in the presentation of L. Senkel: *Computation of Confidence Regions in Reliable, Variable-Structure State and Parameter Estimation*



## Conclusions and Outlook on Future Work

- Verified integration of ODEs with non-smooth right-hand side
- Observer-based identification of parameter intervals
- Consistency with dynamic system model and (uncertain) measured data
- Efficiency of subdivision heuristics
- Validation of alternative estimation procedures (e.g. sliding mode)
- Implementation of parallelization strategies
- Extension to further (higher-dimensional) nonlinear models
- Derivation of optimal input trajectories wrt. exclusion of infeasible intervals

Dziękuję bardzo za uwagę!

Thank you for your attention!

Спасибо за Ваше внимание!

Merci beaucoup pour votre attention!

¡Muchas gracias por su atención!

Grazie mille per la vostra attenzione!

Vielen Dank für Ihre Aufmerksamkeit!

