Universität Rostock

Traditio et Innovatio

Chair of Mechatronics

Numerical Validation of Sliding Mode Approaches with Uncertainty

SCAN 2014: 16th GAMM-IMACS International Symposium on Scientific Computing, Computer Arithmetic and Validated Numerics

Würzburg, Germany

25 September, 2014

Luise Senkel, Andreas Rauh, Harald Aschemann

Chair of Mechatronics
University of Rostock, Germany

Contents

Motivation

Requirements for Robust Control Approaches

- Trajectory tracking despite non-measurable system states
- Noisy processes and measurements
- System models and applications are affected by uncertainty

Motivation

Bounded Uncertainty

- Lack of knowledge about system parameters
- Manufacturing tolerances

Solution

- Consideration of intervals for
  - Parameters and states
  - Measurement, control, and estimation errors

Motivation

Stochastic Uncertainty

- Inaccurate sensor measurements
  $\rightarrow$ Filtering is necessary to obtain smooth signals
- Uncertain or random effects, e.g. friction

Solution

- Consideration of non-modeled influences on the system as process noise (Brownian motion)
- Consideration of additive disturbances on measured data as measurement noise (Brownian motion)
- Itô differential operator and Lyapunov functions for calculation of switching amplitudes in variable-structure control and observer strategies

Motivation

Common Sliding Mode Techniques

Advantages

- Definition of a sliding surface based on a Lyapunov function candidate
- Robustness against uncertain parameters and states
- Finite-time convergence

Disadvantage

- Restrictive matching conditions wrt. the system model
- Chattering due to unnecessarily large predefined switching amplitudes

Motivation

Advanced Sliding Mode Control Procedure

- Robust control strategy considering noise processes
- Intervals for states and uncertain parameters
- Less severe restrictions on the system model
- Suitable candidates of Lyapunov functions for asymptotic stability and calculation of switching amplitudes
- Number of calculated switching amplitudes is equal to system order instead of predefining just one switching amplitude
- Adaptation of switching amplitudes of the controller's variable structure part in every time step
- Implementation using C++ S-functions in MATLAB with software library C-XSC (real-time applicable)

1. Motivation
2. Interval Arithmetics for Control Strategies
3. Robust Sliding Mode Control Approach
4. Illustrative Benchmark Example
5. Simulation Results
6. Experimental Implementation (in Progress)
7. Conclusions and Outlook on Further Work

Interval Definitions for Control Design

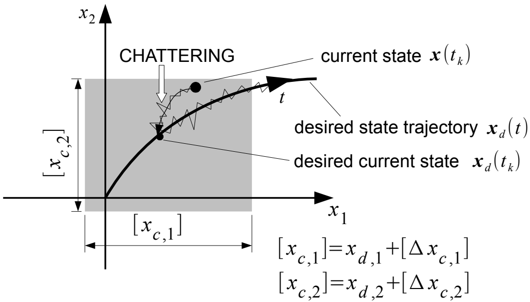


Figure: Location of intervals for sliding mode control design.

Sliding Mode Techniques

Principle

- Definition of a sliding surface $\mathbf{s} = \mathbf{x} - \mathbf{x}_d$ such that the system states tend to this stable mode
- Suitable candidate of a Lyapunov function $V = \frac{1}{2} \cdot \mathbf{s}^T \cdot \mathbf{s} > 0$
- Stabilization of tracking error $\dot{V} = \mathbf{s}^T \cdot \dot{\mathbf{s}} \leq 0$

Sliding Mode Techniques

Principle

- Definition of a sliding surface $\mathbf{s} = \mathbf{x} - \mathbf{x}_d$ such that the system states tend to this stable mode
- Suitable candidate of a Lyapunov function $V = \frac{1}{2} \cdot \mathbf{s}^T \cdot \mathbf{s} > 0$
- Stabilization of tracking error $\dot{V} = \mathbf{s}^T \cdot \dot{\mathbf{s}} \leq 0$

Following Description

- Part 1: Stabilization of the **linear system part by pole placement**

Sliding Mode Techniques

Principle

- Definition of a sliding surface $\mathbf{s} = \mathbf{x} - \mathbf{x}_d$ such that the system states tend to this stable mode
- Suitable candidate of a Lyapunov function $V = \frac{1}{2} \cdot \mathbf{s}^T \cdot \mathbf{s} > 0$
- Stabilization of tracking error $\dot{V} = \mathbf{s}^T \cdot \dot{\mathbf{s}} \leq 0$

Following Description

- Part 1: Stabilization of the **linear system part by pole placement**
- Part 2: Stabilization of the **nonlinear system** part with bounded and stochastic uncertainties by **sliding mode control**

Sliding Mode Control: Scheme

System model, stochastic ODE with interval parameters

$$d\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), [\boldsymbol{p}])dt + \boldsymbol{G}_p d\boldsymbol{w}_p$$
$$\boldsymbol{y} = \boldsymbol{C}_C([\boldsymbol{p}])\boldsymbol{x}(t) + \boldsymbol{G}_m d\boldsymbol{w}_m$$

$$[\boldsymbol{p}] = [\underline{\boldsymbol{p}}\,;\,\overline{\boldsymbol{p}}]$$

Desired state trajectories up to the n-th derivative

$$\boldsymbol{x}_d = [x_{1,d}, x_{2,d}, \ldots, x_{n,d}]^T$$

Sliding Mode Control: Scheme

System model, stochastic ODE with interval parameters

$$d\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), [\boldsymbol{p}])dt + \boldsymbol{G}_p d\boldsymbol{w}_p$$

$$\boldsymbol{y} = \boldsymbol{C}_C([\boldsymbol{p}])\boldsymbol{x}(t) + \boldsymbol{G}_m d\boldsymbol{w}_m$$

$$[\boldsymbol{p}] = [\underline{\boldsymbol{p}} ; \overline{\boldsymbol{p}}]$$

Desired state trajectories up to the n-th derivative

$$\boldsymbol{x}_d = [x_{1,d}, x_{2,d}, \ldots, x_{n,d}]^T$$

Suitable cancidate of a Lyapunov function

$$V_C = \frac{1}{2}(\boldsymbol{x} - \boldsymbol{x}_d)^T \boldsymbol{P}_C (\boldsymbol{x} - \boldsymbol{x}_d)$$

Sliding Mode Control: Scheme

System model, stochastic ODE with interval parameters

$$d\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), [\boldsymbol{p}])dt + \boldsymbol{G}_p d\boldsymbol{w}_p$$

$$\boldsymbol{y} = \boldsymbol{C}_C([\boldsymbol{p}])\boldsymbol{x}(t) + \boldsymbol{G}_m d\boldsymbol{w}_m$$

$$[\boldsymbol{p}] = [\underline{\boldsymbol{p}}\,;\,\overline{\boldsymbol{p}}]$$

Desired state trajectories up to the n-th derivative

$$\boldsymbol{x}_d = [x_{1,d}, x_{2,d}, \dots, x_{n,d}]^T$$

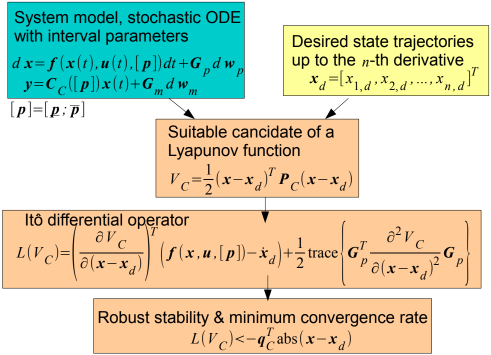
Suitable cancidate of a Lyapunov function

$$V_C = \frac{1}{2}(\boldsymbol{x} - \boldsymbol{x}_d)^T \boldsymbol{P}_C (\boldsymbol{x} - \boldsymbol{x}_d)$$

Itô differential operator

$$L(V_C) = \left(\frac{\partial V_C}{\partial(\boldsymbol{x} - \boldsymbol{x}_d)}\right)^T \left(\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, [\boldsymbol{p}]) - \dot{\boldsymbol{x}}_d\right) + \frac{1}{2}\operatorname{trace}\left\{\boldsymbol{G}_p^T \frac{\partial^2 V_C}{\partial(\boldsymbol{x} - \boldsymbol{x}_d)^2} \boldsymbol{G}_p\right\}$$

Sliding Mode Control: Scheme



Sliding Mode Control: Scheme

System model, stochastic ODE with interval parameters

$$d\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), [\boldsymbol{p}])dt + \boldsymbol{G}_p d\boldsymbol{w}_p$$
$$\boldsymbol{y} = \boldsymbol{C}_C([\boldsymbol{p}])\boldsymbol{x}(t) + \boldsymbol{G}_m d\boldsymbol{w}_m$$

$[\boldsymbol{p}] = [\underline{\boldsymbol{p}}\,;\,\overline{\boldsymbol{p}}]$

Desired state trajectories up to the n-th derivative

$$\boldsymbol{x}_d = [x_{1,d}, x_{2,d}, \ldots, x_{n,d}]^T$$

Suitable cancidate of a Lyapunov function

$$V_C = \frac{1}{2}(\boldsymbol{x}-\boldsymbol{x}_d)^T \boldsymbol{P}_C (\boldsymbol{x}-\boldsymbol{x}_d)$$

Itô differential operator

$$L(V_C) = \left(\frac{\partial V_C}{\partial(\boldsymbol{x}-\boldsymbol{x}_d)}\right)^T \left(\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, [\boldsymbol{p}]) - \dot{\boldsymbol{x}}_d\right) + \frac{1}{2}\,\mathrm{trace}\left\{\boldsymbol{G}_p^T \frac{\partial^2 V_C}{\partial(\boldsymbol{x}-\boldsymbol{x}_d)^2} \boldsymbol{G}_p\right\}$$

Robust stability & minimum convergence rate

$$L(V_C) < -\boldsymbol{q}_C^T \,\mathrm{abs}(\boldsymbol{x}-\boldsymbol{x}_d)$$

Calculation of the switching amplitudes $\boldsymbol{\eta}$

Sliding Mode Control: Scheme

System model, stochastic ODE with interval parameters

$$d\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), [\boldsymbol{p}])dt + \boldsymbol{G}_p d\boldsymbol{w}_p$$
$$\boldsymbol{y} = \boldsymbol{C}_C([\boldsymbol{p}])\boldsymbol{x}(t) + \boldsymbol{G}_m d\boldsymbol{w}_m$$

$[\boldsymbol{p}] = [\underline{\boldsymbol{p}}\,;\,\overline{\boldsymbol{p}}]$

Desired state trajectories up to the n-th derivative

$$\boldsymbol{x}_d = [x_{1,d}, x_{2,d}, \ldots, x_{n,d}]^T$$

Suitable cancidate of a Lyapunov function

$$V_C = \frac{1}{2}(\boldsymbol{x} - \boldsymbol{x}_d)^T \boldsymbol{P}_C (\boldsymbol{x} - \boldsymbol{x}_d)$$

Itô differential operator

$$L(V_C) = \left(\frac{\partial V_C}{\partial(\boldsymbol{x} - \boldsymbol{x}_d)}\right)^T \left(\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, [\boldsymbol{p}]) - \dot{\boldsymbol{x}}_d\right) + \frac{1}{2}\,\mathrm{trace}\left\{\boldsymbol{G}_p^T \frac{\partial^2 V_C}{\partial(\boldsymbol{x} - \boldsymbol{x}_d)^2} \boldsymbol{G}_p\right\}$$

Robust stability & minimum convergence rate

$$L(V_C) < -\boldsymbol{q}_C^T \,\mathrm{abs}(\boldsymbol{x} - \boldsymbol{x}_d)$$

Calculation of the switching amplitudes $\boldsymbol{\eta}$

Control law in $\boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), [\boldsymbol{p}])$

$$\boldsymbol{u} = \boldsymbol{u}_V - \boldsymbol{K}\boldsymbol{x} + \boldsymbol{\eta}^T \mathrm{sign}(\boldsymbol{x} - \boldsymbol{x}_d)$$

Calculation of the Switching Amplitude

- Employing the condition $L(V_C(t)) \overset{!}{<} -\mathbf{q}_C^T \mathrm{abs}\left(\tilde{\mathbf{x}}\right)$ (user-defined convergence rate vector $\mathbf{q}_C > \mathbf{0}$) leads to component-wise calculation of the switching amplitude vector (single-input single-output systems)

$$\eta_i = \begin{cases} \sup\left(\mathbf{M}_i^+ \cdot \left(-\dot{V}_{a,C} - \mathbf{q}_C^T \mathrm{abs}\left(\tilde{\mathbf{x}}\right) - T\right)\right) + \mu, & \mathbf{M}_i^+ < 0 \\ \inf\left(\mathbf{M}_i^+ \cdot \left(-\dot{V}_{a,C} - \mathbf{q}_C^T \mathrm{abs}\left(\tilde{\mathbf{x}}\right) - T\right)\right) - \mu, & \mathbf{M}_i^+ > 0 \\ 0, & \text{else} \end{cases}$$

Calculation of the Switching Amplitude

- Employing the condition $L(V_C(t)) \overset{!}{<} -\mathbf{q}_C^T \text{abs}\left(\tilde{\mathbf{x}}\right)$ (user-defined convergence rate vector $\mathbf{q}_C > \mathbf{0}$) leads to component-wise calculation of the switching amplitude vector (single-input single-output systems)

$$\eta_i = \begin{cases} \sup\left(\mathbf{M}_i^+ \cdot \left(-\dot{V}_{a,C} - \mathbf{q}_C^T \text{abs}\left(\tilde{\mathbf{x}}\right) - T\right)\right) + \mu, & \mathbf{M}_i^+ < 0 \\ \inf\left(\mathbf{M}_i^+ \cdot \left(-\dot{V}_{a,C} - \mathbf{q}_C^T \text{abs}\left(\tilde{\mathbf{x}}\right) - T\right)\right) - \mu, & \mathbf{M}_i^+ > 0 \\ 0, & \text{else} \end{cases}$$

- Substitution $\mathbf{M} = \mathbf{B}_C^T \mathbf{P}_C \left|\tilde{\mathbf{x}}\right|$
- Left pseudo inverse of a matrix $\mathbf{M}$ is denoted by $\mathbf{M}^+ = \left(\mathbf{M}^T \cdot \mathbf{M}\right)^{-1} \cdot \mathbf{M}^T$ (SISO case, $\mathbf{M}$ turns out to be a column vector)

Calculation of the Switching Amplitude

- Substitution $\mathbf{M} = \mathbf{B}_C^T \mathbf{P}_C \, |\tilde{\mathbf{x}}|$
- Left pseudo inverse of a matrix $\mathbf{M}$ is denoted by $\mathbf{M}^{+} = \left(\mathbf{M}^T \cdot \mathbf{M}\right)^{-1} \cdot \mathbf{M}^T$ (SISO case, $\mathbf{M}$ turns out to be a column vector)
- Matrix of the absolute values $|\tilde{\mathbf{x}}| \in \mathbb{R}^{n \times n}$ is defined with the tracking error $\tilde{x}_i = (x_i - x_{d,i})$ (for all $i \in \{1, ..., n\}$) as

$$|\tilde{\mathbf{x}}| = \begin{bmatrix} \tilde{x}_1 \cdot \mathrm{sign}(\tilde{x}_1) & \tilde{x}_1 \cdot \mathrm{sign}(\tilde{x}_2) & \dots & \tilde{x}_1 \cdot \mathrm{sign}(\tilde{x}_n) \\ \tilde{x}_2 \cdot \mathrm{sign}(\tilde{x}_1) & \tilde{x}_2 \cdot \mathrm{sign}(\tilde{x}_2) & \dots & \tilde{x}_2 \cdot \mathrm{sign}(\tilde{x}_n) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_n \cdot \mathrm{sign}(\tilde{x}_1) & \tilde{x}_n \cdot \mathrm{sign}(\tilde{x}_2) & \dots & \tilde{x}_n \cdot \mathrm{sign}(\tilde{x}_n) \end{bmatrix}$$

- Sign function $\mathrm{sign}(\tilde{x}_i) = \begin{cases} 1 & \text{if } \tilde{x}_i > 0 \\ -1 & \text{if } \tilde{x}_i < 0 \\ 0 & \text{else} \end{cases}$

Calculation of the Switching Amplitude

- Employing the condition $L(V_C(t)) \overset{!}{<} -\mathbf{q}_C^T \text{abs}\left(\tilde{\mathbf{x}}\right)$ (user-defined convergence rate vector $\mathbf{q}_C > \mathbf{0}$) leads to component-wise calculation of the switching amplitude vector (single-input single-output systems)

$$\eta_i = \begin{cases} \sup\left(\mathbf{M}_i^+ \cdot \left(-\dot{V}_{a,C} - \mathbf{q}_C^T \text{abs}\left(\tilde{\mathbf{x}}\right) - T\right)\right) + \mu, & \mathbf{M}_i^+ < 0 \\ \inf\left(\mathbf{M}_i^+ \cdot \left(-\dot{V}_{a,C} - \mathbf{q}_C^T \text{abs}\left(\tilde{\mathbf{x}}\right) - T\right)\right) - \mu, & \mathbf{M}_i^+ > 0 \\ 0, & \text{else} \end{cases}$$

- Linear part of the Lyapunov function with feedforward control u_V results in $\dot{V}_{a,C} = \tilde{\mathbf{x}}^T \mathbf{P}_C (\mathbf{A}_C - \mathbf{B}_C \mathbf{K})\mathbf{x} + \tilde{\mathbf{x}}^T \mathbf{P}_C \mathbf{B}_C u_V - \tilde{\mathbf{x}}^T \mathbf{P}_C \dot{\mathbf{x}}_d$
- Vector of absolute values (component-wise) $\text{abs}\left(\tilde{\mathbf{x}}\right) = \begin{bmatrix} \text{abs}(x_1 - x_{d,1}) & \dots & \text{abs}(x_n - x_{d,n}) \end{bmatrix}^T$
- Trace $T = \frac{1}{2} \text{trace}\left\{ \mathbf{G}_p^T \frac{\partial^2 V_C}{\partial \tilde{\mathbf{x}}^2} \mathbf{G}_p \right\}$

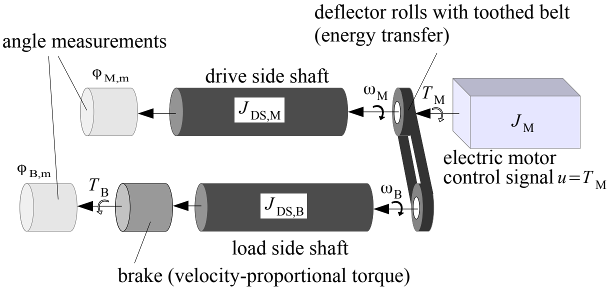
Calculation of the Switching Amplitude

- Employing the condition $L(V_C(t)) \overset{!}{<} -\mathbf{q}_C^T \mathrm{abs}(\tilde{\mathbf{x}})$ (user-defined convergence rate vector $\mathbf{q}_C > \mathbf{0}$) leads to component-wise calculation of the switching amplitude vector (single-input single-output systems)

$$\eta_i = \begin{cases} \sup\left(\mathbf{M}_i^+ \cdot \left(-\dot{V}_{a,C} - \mathbf{q}_C^T \mathrm{abs}(\tilde{\mathbf{x}}) - T\right)\right) + \mu, & \mathbf{M}_i^+ < 0 \\ \inf\left(\mathbf{M}_i^+ \cdot \left(-\dot{V}_{a,C} - \mathbf{q}_C^T \mathrm{abs}(\tilde{\mathbf{x}}) - T\right)\right) - \mu, & \mathbf{M}_i^+ > 0 \\ 0, & \text{else} \end{cases}$$

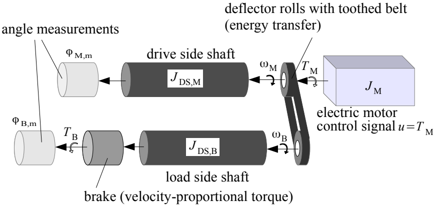
- Underlying stabilization of the error dynamics by the controller gain matrix $\mathbf{K}$ (pole assignment, minimizing a quadratic cost function)
- Matrix $\mathbf{P}_C$ results from solving the Lyapunov equation $\tilde{\mathbf{A}}^T \cdot \mathbf{P}_C + \mathbf{P}_C^T \cdot \tilde{\mathbf{A}} + \mathbf{Q} = \mathbf{0}$ with $\tilde{\mathbf{A}} = \mathbf{A} - \mathbf{B} \cdot \mathbf{K}$ and $\mathbf{Q} > 0$
- Online evaluation of the switching amplitudes η in each time step instead of predefining the switching amplitude

Application Scenario: Drive train test-rig



- Motor torque T_M, braking torque T_B
- Angular velocity of the motor ω_M
- Measured angles $\varphi_{M,m}$ as well as $\varphi_{B,m}$
- J_{rot} contains all mass moments of inertia $J_{DS,M}$, $J_{DS,B}$, J_M with respect to the driving shaft
- Braking represents a disturbance

Application Scenario: Drive Train Test-Rig



System model

- ODE $J_{rot} \cdot \dot{\omega}_M = T_M - T_B + T_S \cdot \mathrm{sign}(\omega_M)$
- Compensation of static friction $T_S \cdot \mathrm{sign}(\omega_{M,d})$
- Motor torque is controlled by interval-based sliding mode controller
- Transmission ratio $k = \frac{\omega_M}{\omega_B}$
- Braking torque $T_B = k_{D_2} \cdot \omega_B = \frac{k_{D_2}}{k} \cdot \omega_M = d \cdot \omega_M$

Application Scenario: Drive train test-rig

System Model (φ_M angle of rotation of the motor shaft)

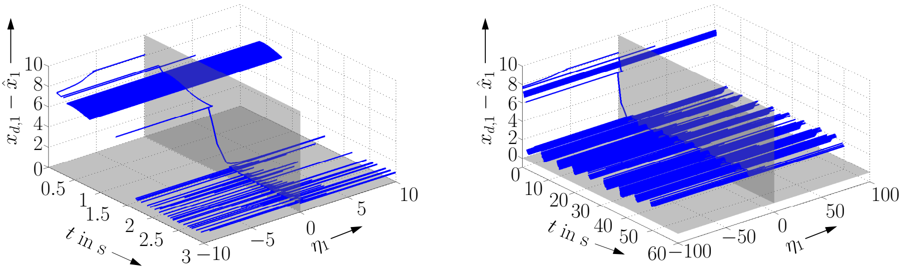
$$\mathbf{f}_N = \begin{bmatrix} \dot{x}_{N1} \\ \dot{x}_{N2} \end{bmatrix} = \begin{bmatrix} \dot{\varphi}_M \\ \dot{\omega}_M \end{bmatrix} = \begin{bmatrix} \omega_M \\ \alpha \cdot \omega_M + \beta \cdot u \end{bmatrix} \text{ with } u = T_M - T_S \cdot \operatorname{sign}(\omega_M)$$

Task for Interval-based Sliding Mode Control

- Trajectory tracking $\varphi_M - \varphi_{M,d} \stackrel{!}{=} 0$ and $\omega_M - \omega_{M,d} \stackrel{!}{=} 0$ despite uncertain parameters $\alpha \in [\alpha]$ and $\beta \in [\beta]$
- Simulative implementation: Interface between MATLAB/SIMULINK and C-XSC
- Experimental implementation: in progress (later more)

Simulation Results - Only Swiching Part in Control Law

$$u = \eta^T \mathrm{sign}(\mathbf{x} - \mathbf{x}_d)$$



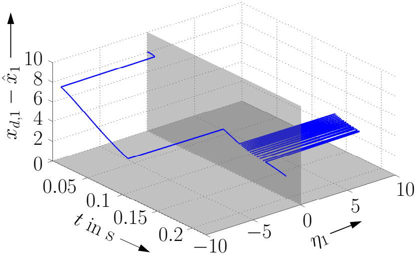
(a) Transient phase. (b) Complete time horizon.

Relation between tracking error of position and corresponding switching amplitude

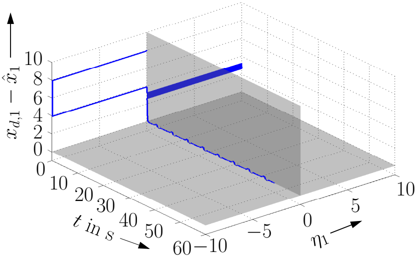
Tracking error after transient phase approx. $x_{d,1} - \hat{x}_1 = [0.9; 1.9]$

Simulation Results - Complete Control Law

$$u = u_V - \mathbf{k}^T \cdot \mathbf{x} + \eta^T \mathrm{sign}(\mathbf{x} - \mathbf{x}_d)$$



(c) Transient phase.



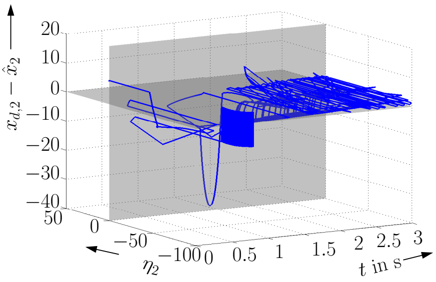
(d) Complete time horizon

Relation between tracking error of position and corresponding switching amplitude

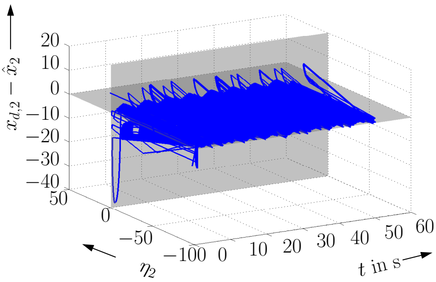
Tracking error after transient phase approx. $x_{d,1} - \hat{x}_1 = [-0.1; 0.2]$

Simulation Results - Only Switching Part in Control Law

$u = \eta^T \mathrm{sign}(\mathbf{x} - \mathbf{x}_d)$



(e) Transient phase.



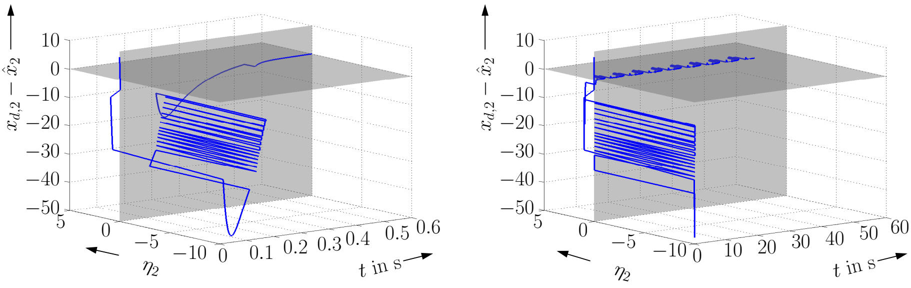
(f) Complete time horizon.

Relation between tracking error of velocity and corresponding switching amplitude

Tracking error after transient phase approx. $x_{d,2} - \hat{x}_2 = [-10; 10]$

Simulation Results - Complete Control Law

$$u = u_V - \mathbf{k}^T \cdot \mathbf{x} + \eta^T \mathrm{sign}(\mathbf{x} - \mathbf{x}_d)$$



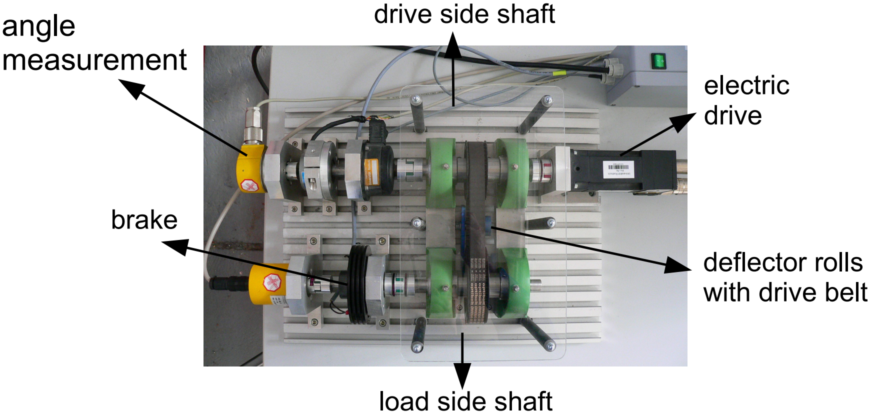
(g) Transient phase.

(h) Complete time horizon

Relation between tracking error of velocity and corresponding switching amplitude

Tracking error after transient phase approx. $x_{d,2} - \hat{x}_2 = [-2; 1]$

Experimental Implementation (in Progress)



Experimental Test-Rig

Hardware and Programming

- Real-time environment: Bachmann system
- Communication via serial interface and Ethernet
- Compilation of Matlab/Simulink model directly on process unit
- Interface with C-XSC does not work in experiment, because switching of rounding mode is not fully supported (loss of accuracy can be neglected)
  $\Rightarrow$ Own structure for calculating with intervals
  $\Rightarrow$ New implementation of all algebraic operators
- Implementation of algorithm is in progress

Experimental Test-Rig

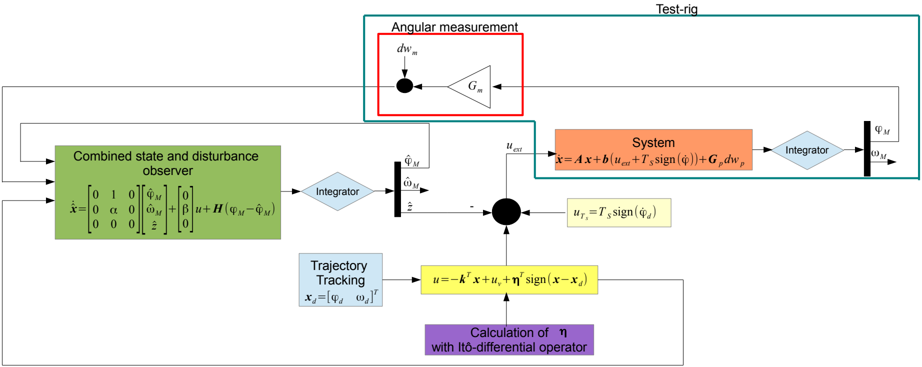


Figure: Schematic overview of the implementation in experiment.

Experimental Test-Rig

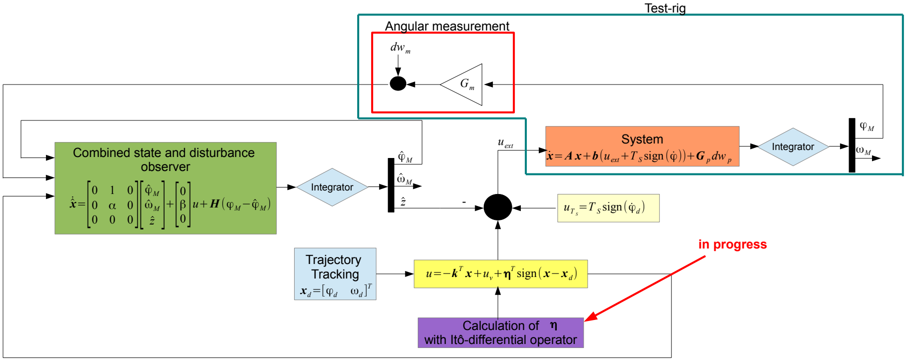


Figure: Schematic overview of the implementation in experiment.

Conclusions and Outlook

Conclusions

- Development of a robust interval-based sliding mode controller
- Validation in simulation

Conclusions and Outlook

Conclusions

- Development of a robust interval-based sliding mode controller
- Validation in simulation

Outlook

- Finish experimental implementation for real-time target
- Application to other nonlinear systems: thermodynamic process of a fuel cell system
- Maximization of the guaranteed stabilizable domain of the Lyapunov function
- Combination with interval-based sliding mode observer (estimation of non-measurable states and identification of uncertain parameters) $\Rightarrow$ closed-loop control

Thank you for your attention!