



Analytic Performance Evaluation of the RED Algorithm

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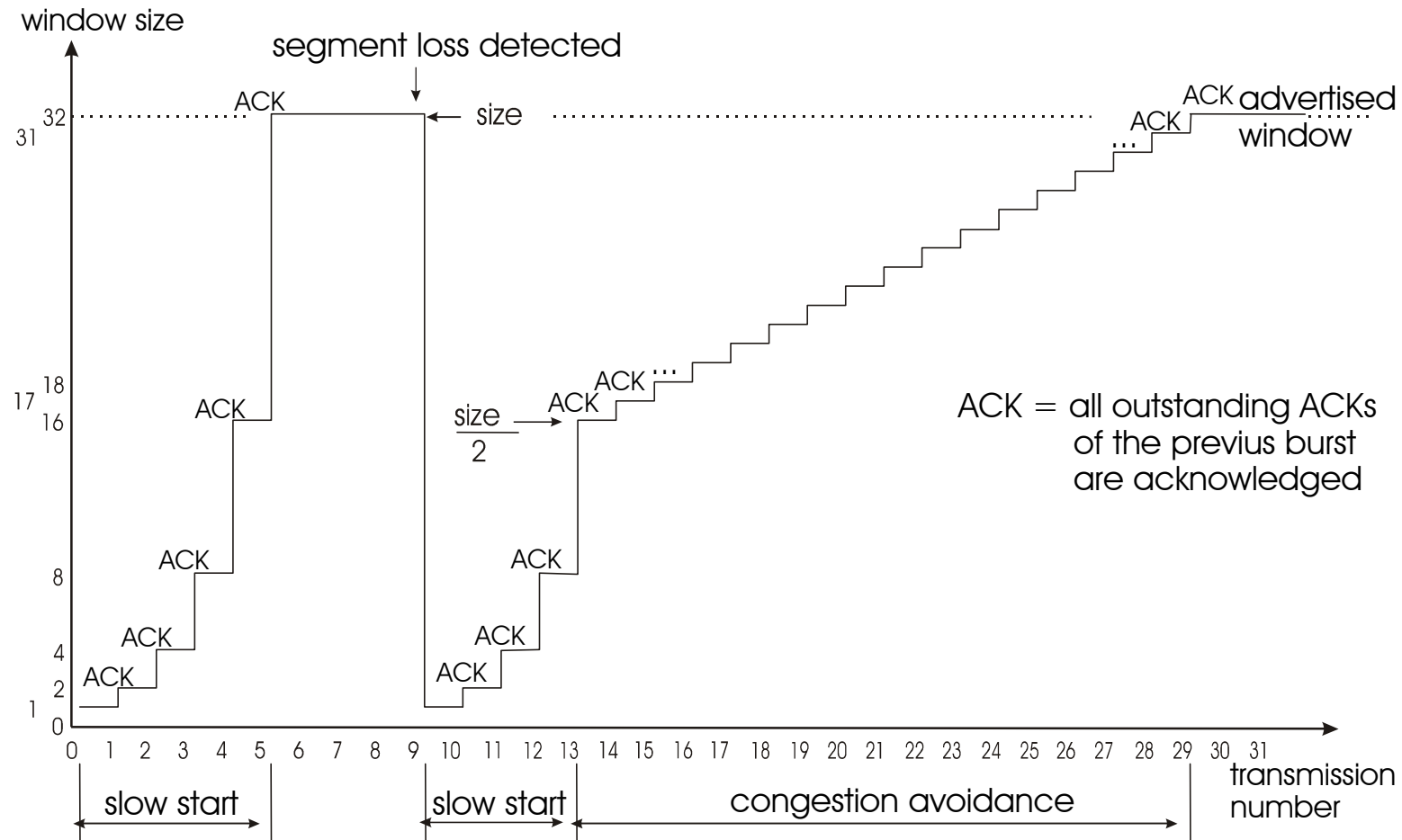
- TCP Model
- RED Model
- TCP over RED
- Results

TCP

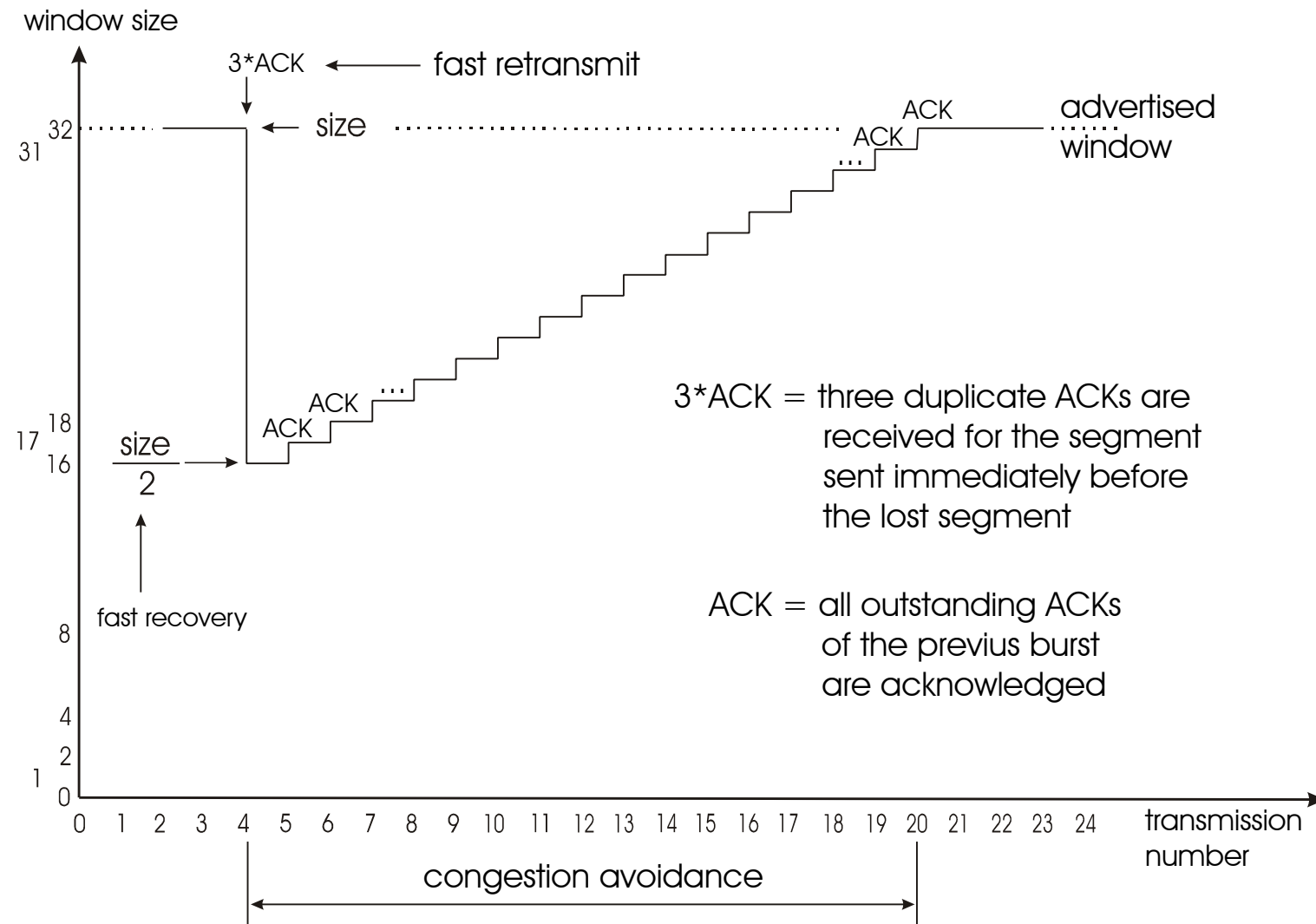
- ▷ Reliable transmission
- ▷ Closed loop flow control
- ▷ Elastic traffic without real-time requirements
- ▷ Major part of Internet traffic
- ▷ Interactive Applications



TCP's slow start and congestion avoidance

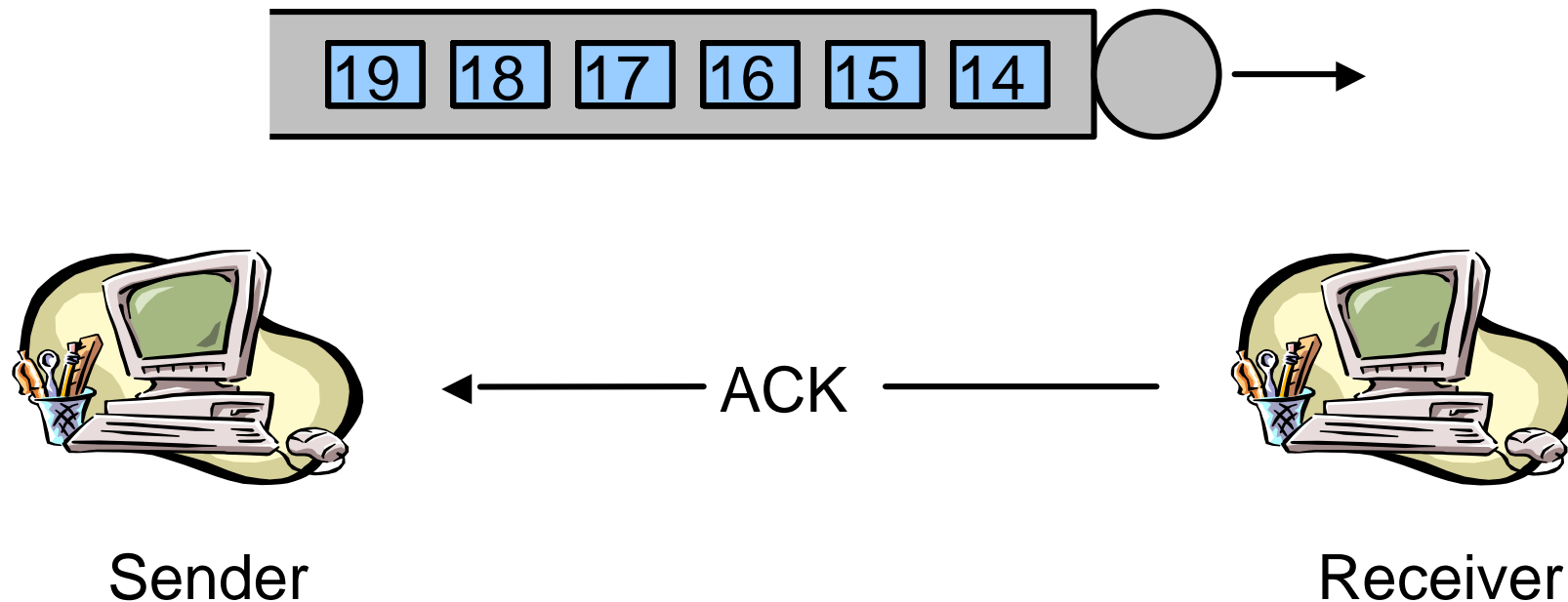


TCP's fast retransmit algorithm

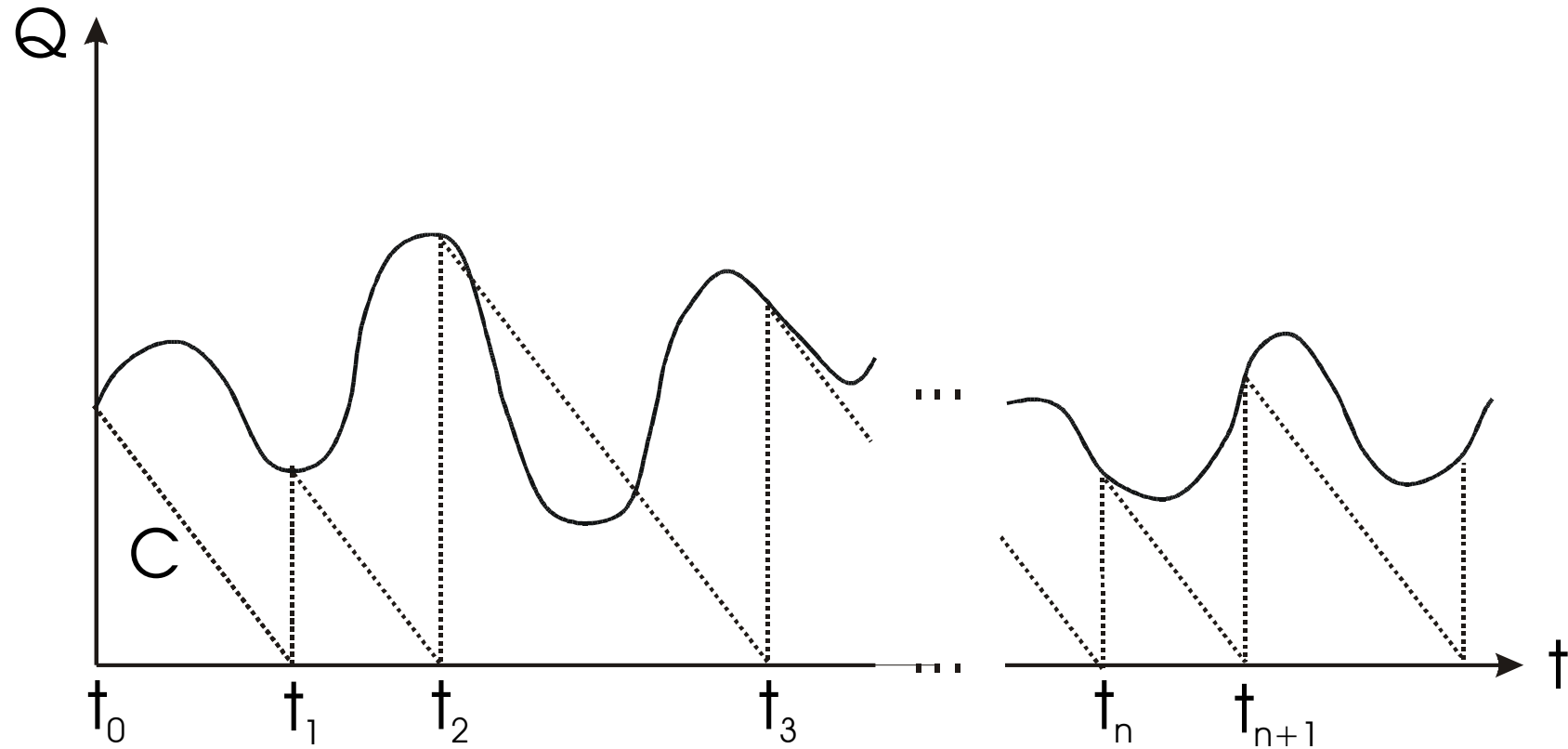


Analytical Approach

- ▷ Semi-Markov process (SMP)
- ▷ Renewal points: Time instant when last packet that has seen the last renewal point has left the queue.
- ▷ „TCP rounds“



Renewal Points



Discrete-Time Model for a Single TCP Connection

▷ Assumptions

- TCP Reno
- FTP-source
- Independent packet losses (later based on RED)
- State variables:
 $(W_n, S_n, M_n) := (\text{CWND}(t), \text{SSTRESH}(t), \text{Loss}(t))$
- Observation: TCP-Round



A Single TCP Connection

Input: model state (W_n, S_n, M_n) , model factor $(L(W_n))$

```
if ( $M_n = 0$ ) then           {no loss last round}
   $S_{n+1} := S_n$ 
  if ( $W_n = W_{\max}$ ) then     {full window possible}
     $W_{n+1} := W_n$ 
  else
    if ( $W_n < S_n$ ) then     {slow start}
       $W_{n+1} := 2 * W_n$ 
    else                     {congestion avoidance}
       $W_{n+1} := W_n + 1$ 
    endif
  endif
endif
else
```



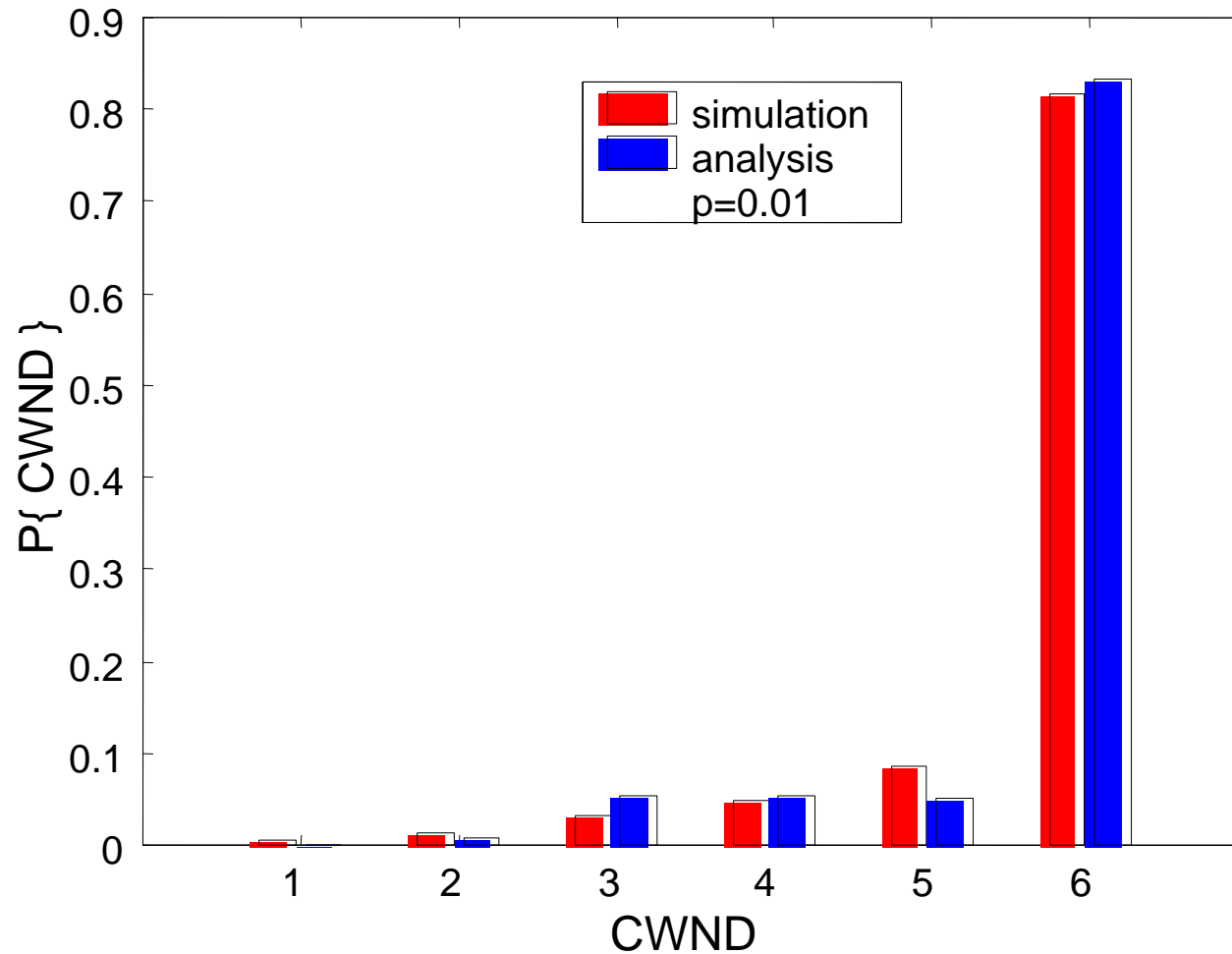
A Single TCP Connection

```
else
  if ( $M_n = 1$ ) then           {one loss last round}
     $S_{n+1} := \max(W_n/2, 2)$ 
     $W_{n+1} := S_{n+1}$ 
  else                         {more than one loss last round}
     $S_{n+1} := \max(W_n/2, 2)$ 
     $W_{n+1} := 1$ 
  endif
endif
 $M_{n+1} := \min(L(W_n), 2)$ 
```

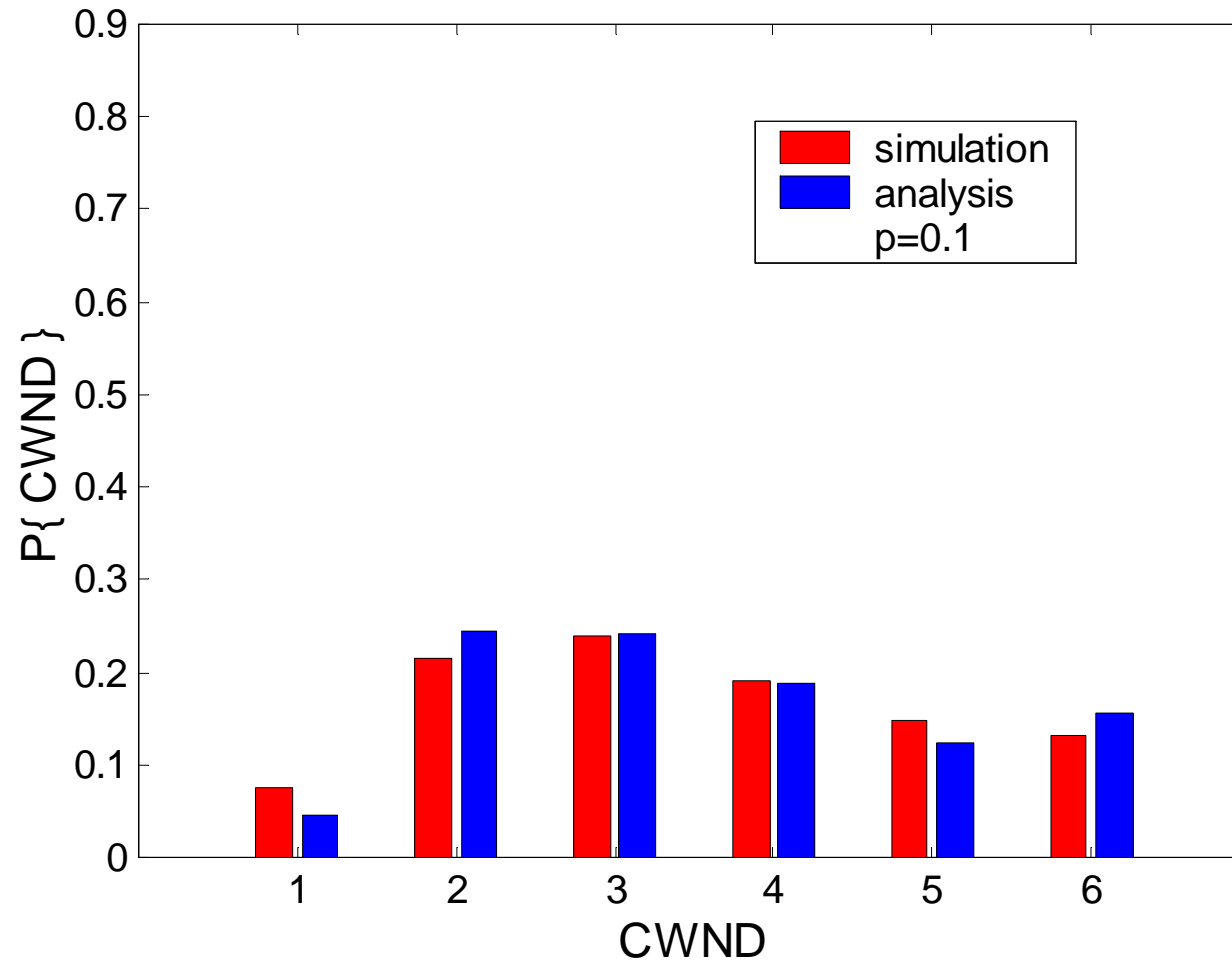
Output: model state($W_{n+1}, S_{n+1}, M_{n+1}$)



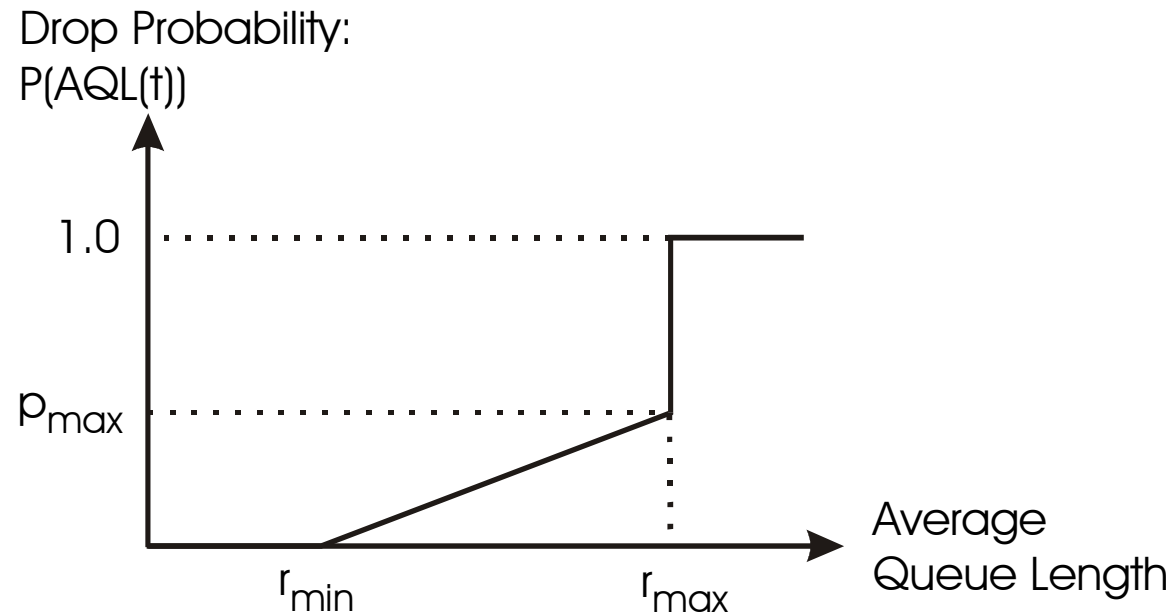
Simulation \Leftrightarrow Analysis (I)



Simulation \Leftrightarrow Analysis (II)



Random Early Discard (RED) Queue



$$p(A=i) = \begin{cases} 0 & 0 \leq i < r_{\min} \\ \left(\frac{i - r_{\min}}{r_{\max} - r_{\min}} \right) \cdot p_{\max} & r_{\min} \leq i \leq r_{\max} \\ 1 & r_{\max} < i \leq \infty \end{cases}$$



RED Queue Mechanism

Input: model state (A_n), model factor ($B, L(B, A_n)$)

$$Q := B - L(B, A_n)$$

$$A_{n+1} := w_q \cdot Q + (1 - w_q) \cdot A_n$$

B: batch of packets
A: average queue size
L: loss
Q: actual queue size
 w_q : weighting factor

Output: model state (A_{n+1})

The probability of k losses within a batch B of j packets follows a binomial distribution:

$$l(A = i, B = j)[k] = \binom{j}{k} p(A = i)^k \cdot (1 - p(A = i))^{j-k}$$



Compound Analysis

▷ h TCP Connections under RED:

Input: model state $((W_{i_n}, S_{i_n}, M_{i_n}), (A_n))$, model factor $Li(A_n, W_{i_n})$

for $i \in \hat{I} \setminus \{1, \dots, h\}$ **do**
 $(W_{i_{n+1}}, S_{i_{n+1}}, M_{i_{n+1}}) := TCP((W_{i_n}, S_{i_n}, M_{i_n}), Li(A_n, W_{i_n}))$
end for

$A_{n+1} := RED((A_n), (S W_{i_n}, S Li(A_n, W_{i_n})))$

Output: model state $((W_{i_{n+1}}, S_{i_{n+1}}, M_{i_{n+1}}), (Q_{n+1}, A_{n+1}))$





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Results

Parameters

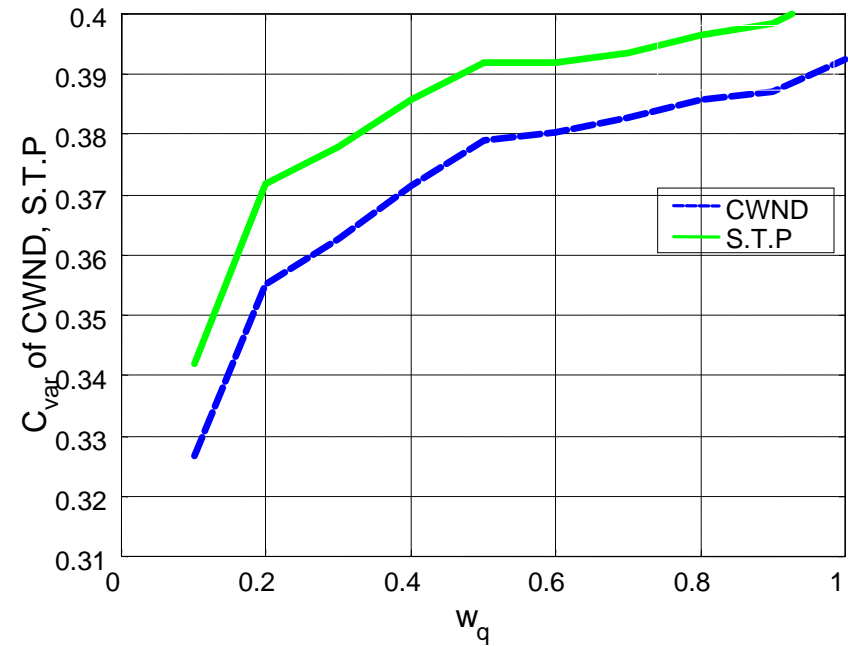
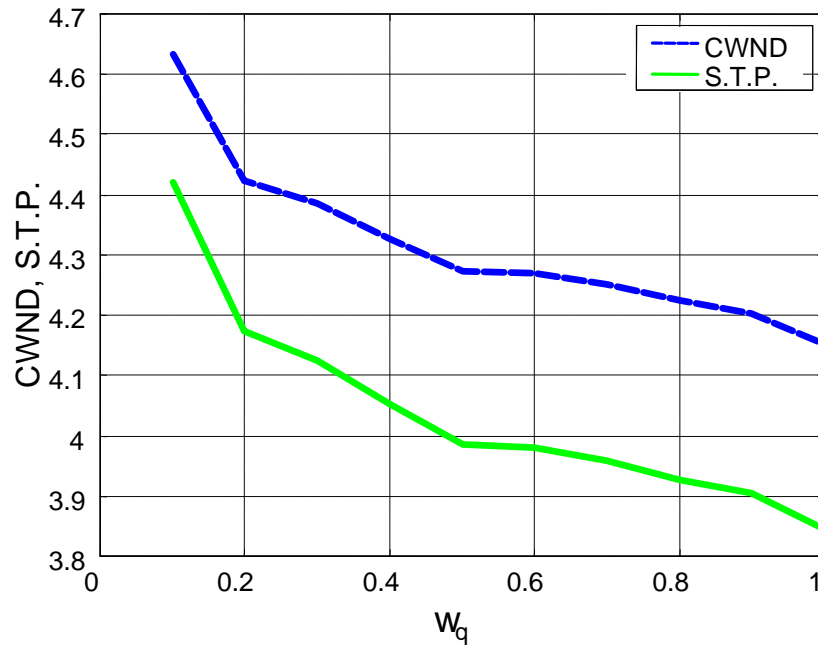
If not stated differently

- $R_{\min}=9$
- $R_{\max}=18$
- $w_q=0.3$
- 3 TCP sources
- CWND=6



Influence of the Weighting Factor

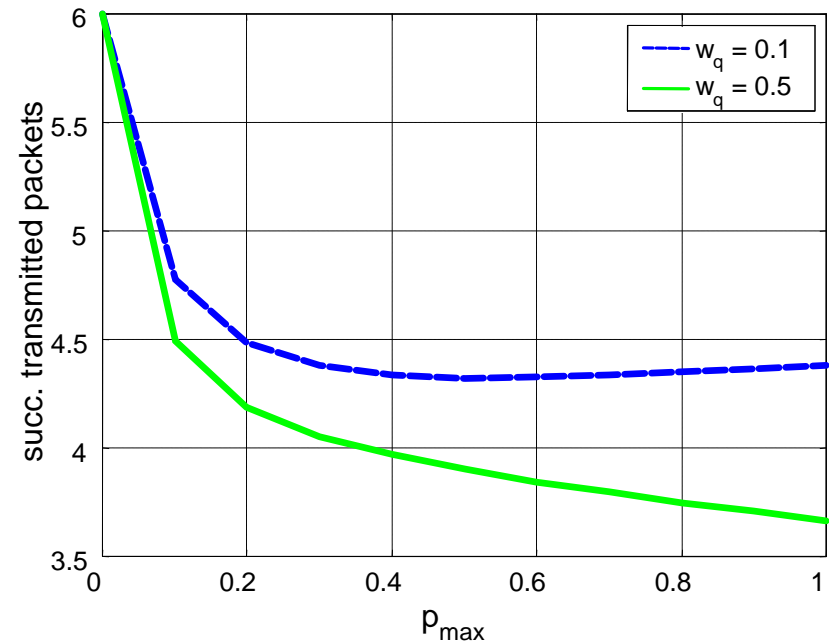
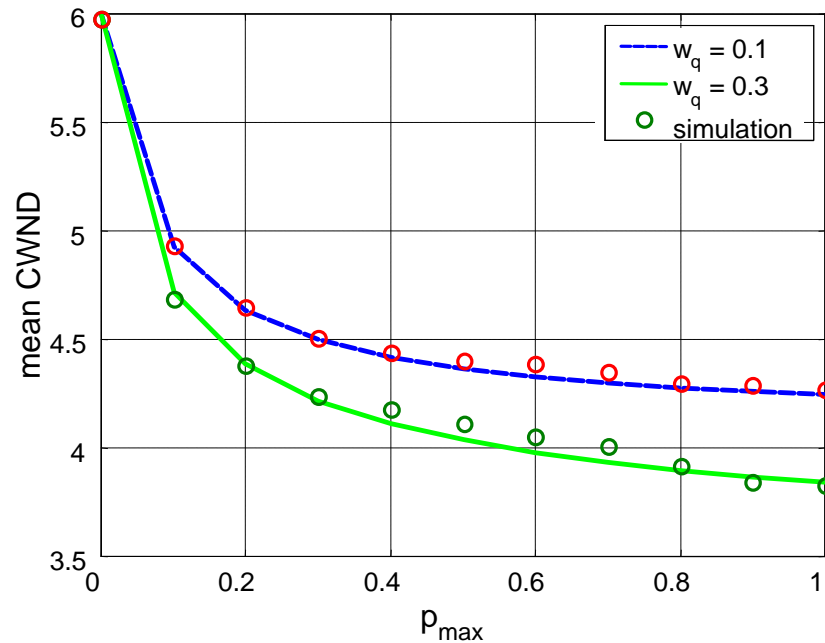
$$p_{\max} = 0.5$$



▷ Increased throughput, reduced variance



Influence of the Loss Function

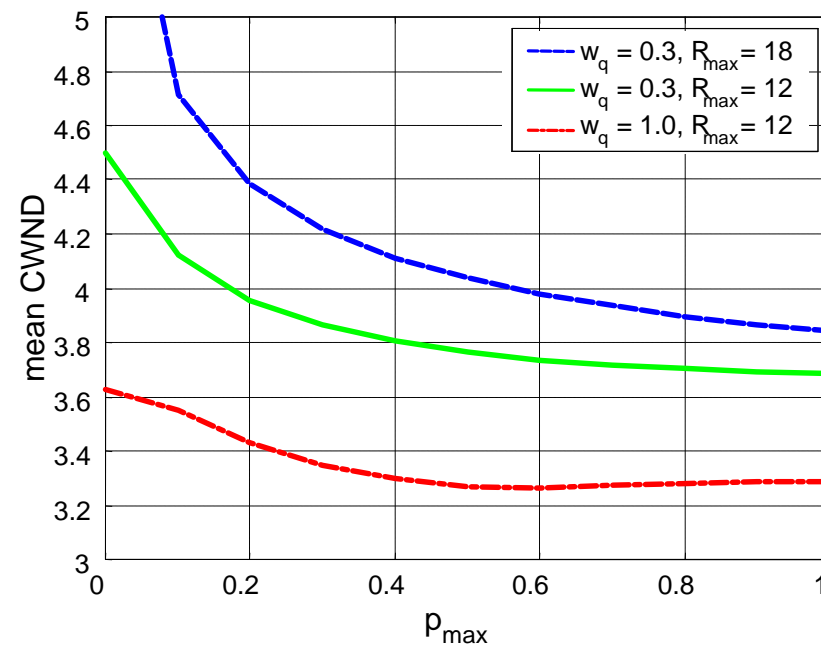
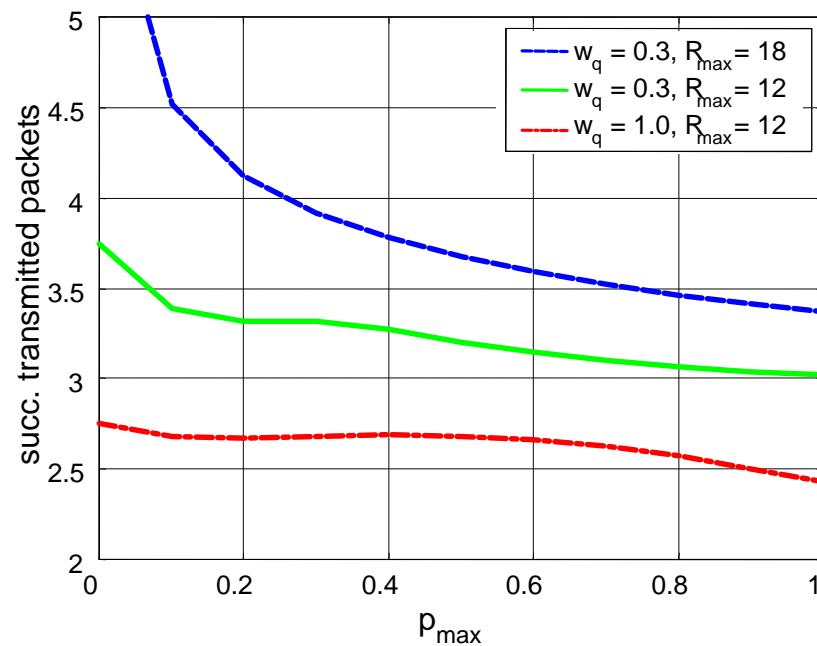


- ▷ High loss probability reduces throughput.
- ▷ “Long Memory” provides better results.



Influence of the Buffer Size

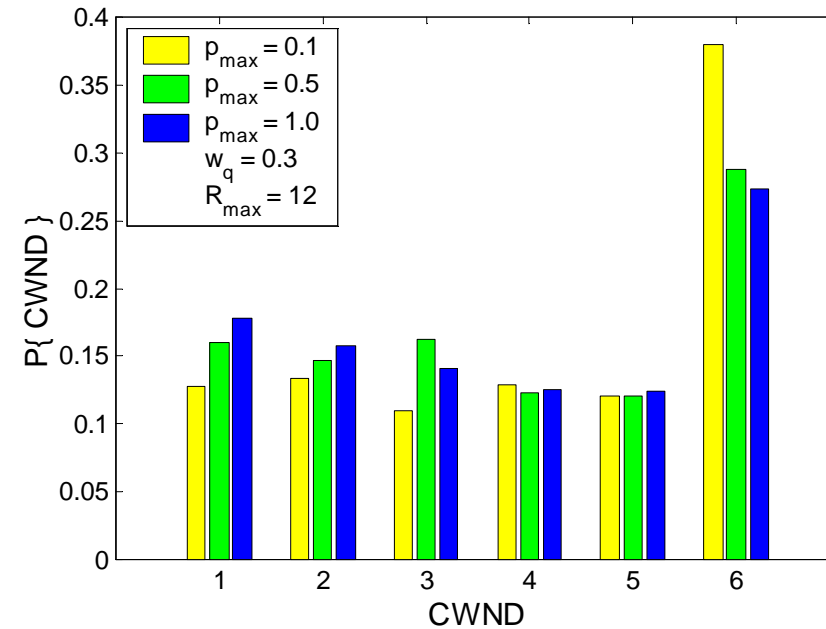
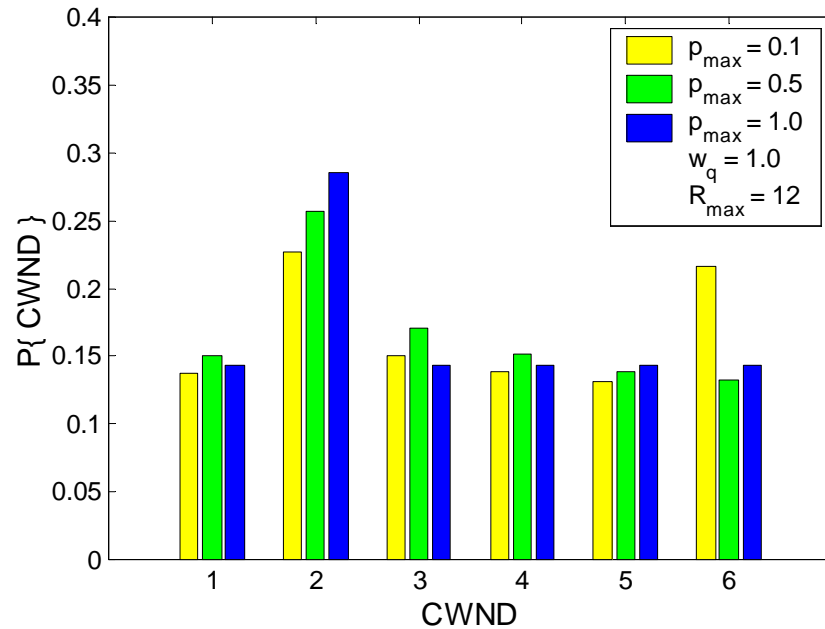
$w_q = 0.3$



- ▷ Short (or congested) queues limit the congestion window size.
- ▷ FIFO \Leftrightarrow RED



Distribution of CWND



- ▷ “Long Memory” and small loss probabilities show larger congestion windows



Conclusion and Outlook

▷ Summary

- Discrete-Time Model of TCP and RED
 - Correlation of TCP sources
- Analytical Performance Evaluation
- Distributions for all TCP state variables

▷ Results

- Sensitivity of TCP to
 - Weighting factor
 - Loss function
 - Congestion or short queues
- Good Performance: “Smoothed FIFO-Queues”

▷ Outlook

- Comparison of different TCP implementations
- Influence of non-linear loss functions
- Fairness studies





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THE END
