



---

# Analytic Performance Evaluation of the RED Algorithm

---

Stefan Köhler, Michael Menth, Norbert Vicari

- TCP Model
- RED Model
- TCP over RED
- Results

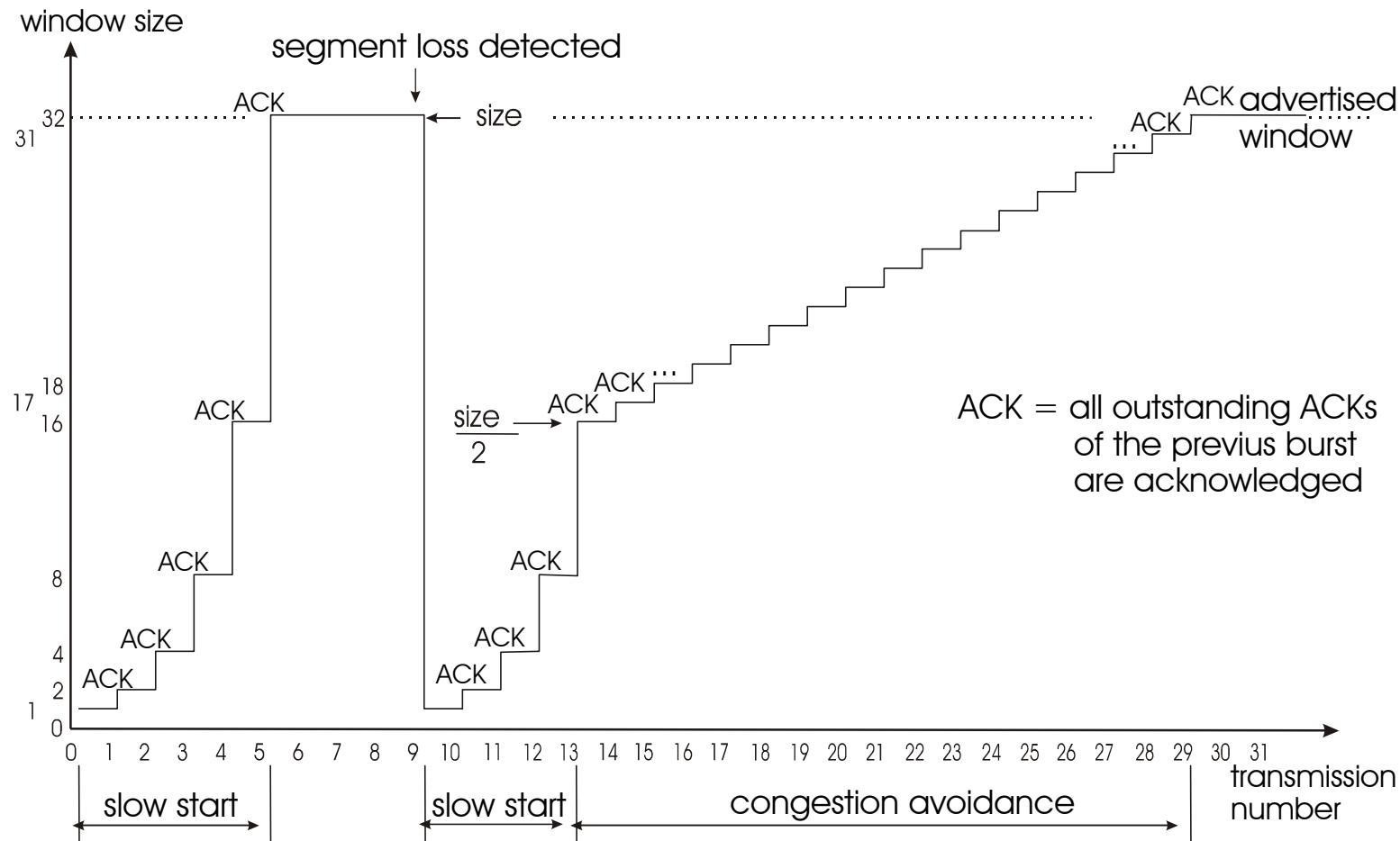
# TCP

---

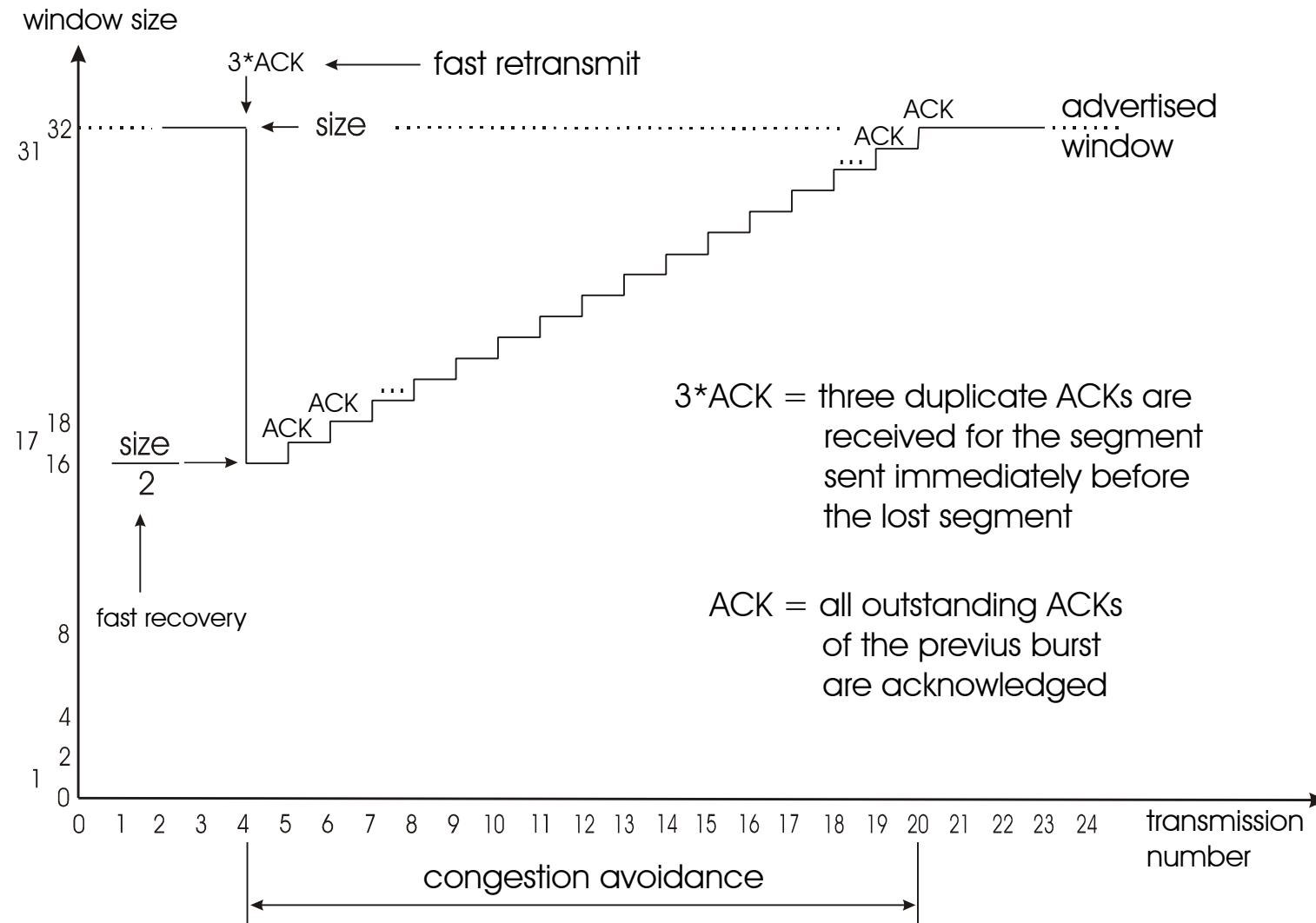
- ▷ Reliable transmission
- ▷ Closed loop flow control
- ▷ Elastic traffic without real-time requirements
- ▷ Major part of Internet traffic
- ▷ Interactive Applications



# TCP's slow start and congestion avoidance



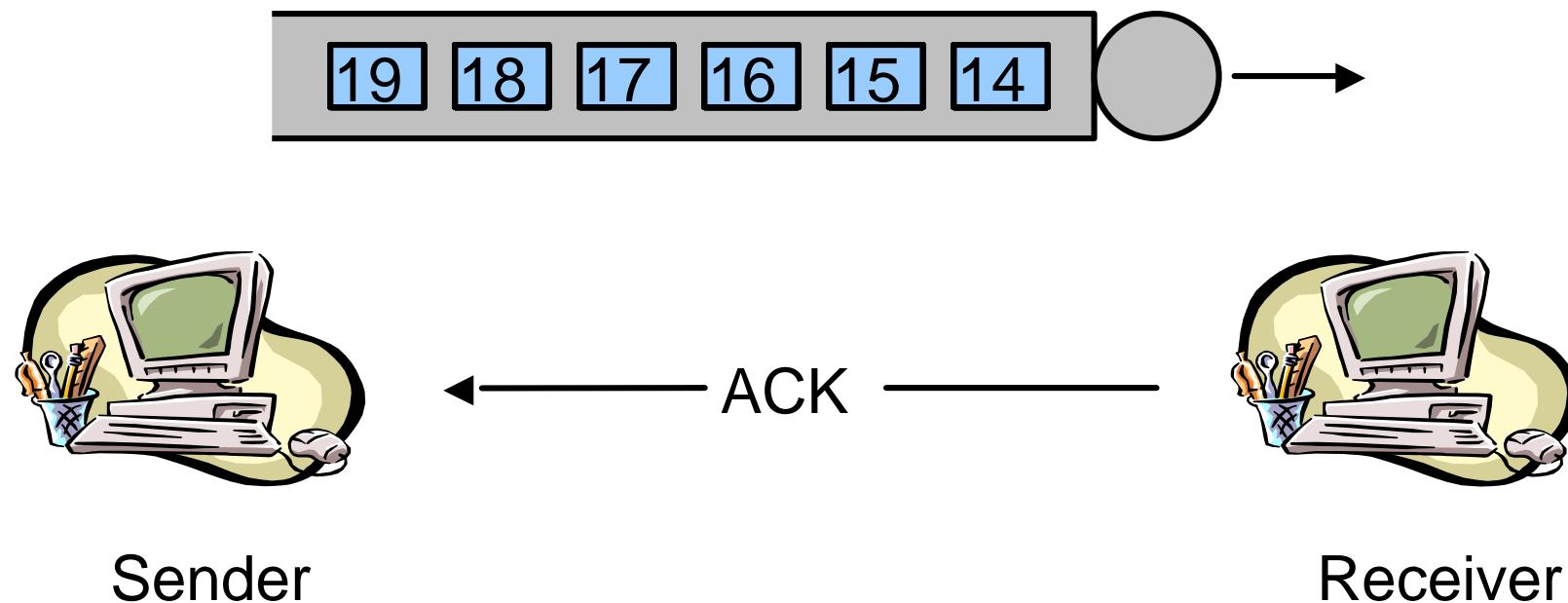
# TCP's fast retransmit algorithm



# Analytical Approach

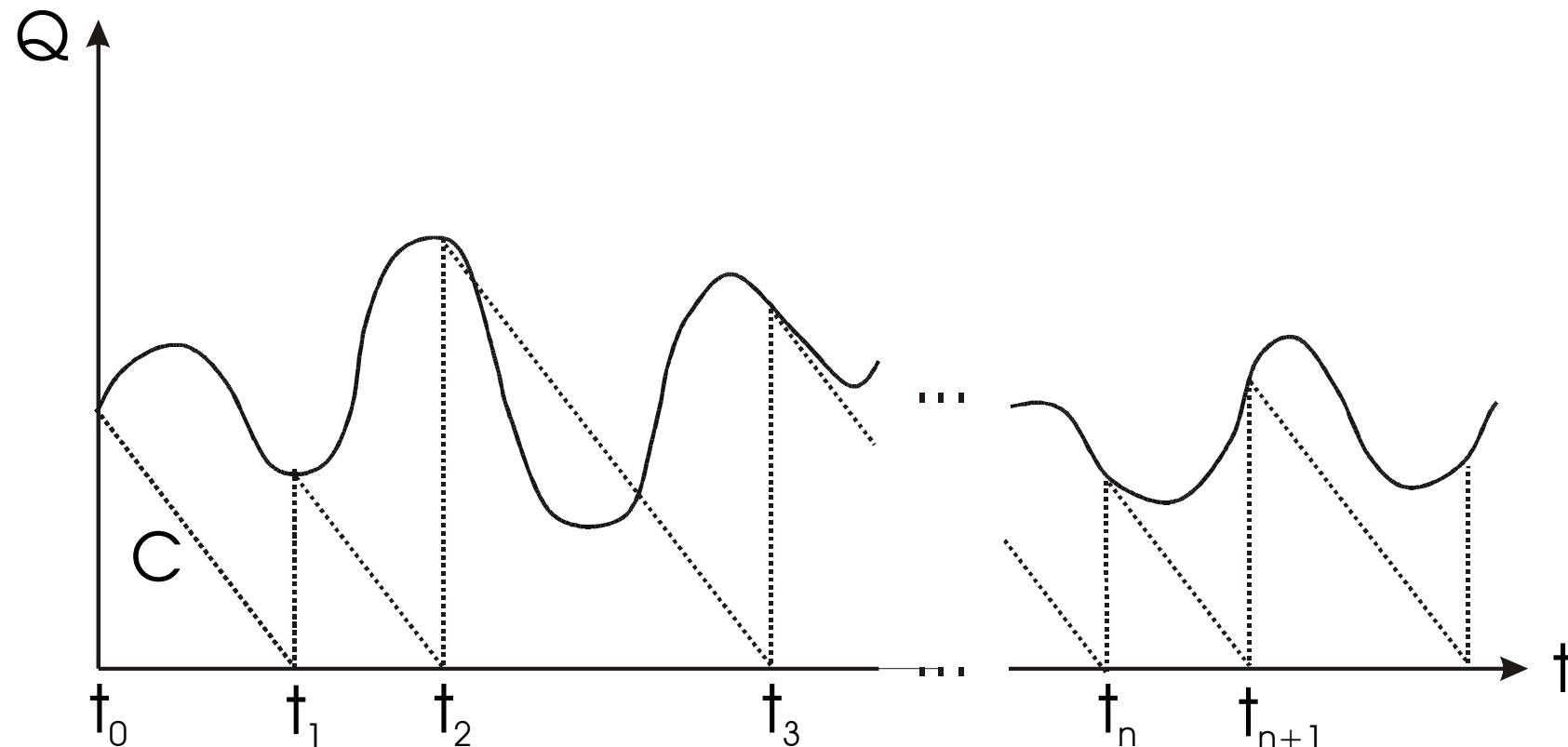
---

- ▷ Semi-Markov process (SMP)
- ▷ Renewal points: Time instant when last packet that has seen the last renewal point has left the queue.
- ▷ „TCP rounds“



# Renewal Points

---



# Discrete-Time Model for a Single TCP Connection

---

## ▷ Assumptions

- TCP Reno
- FTP-source
- Independent packet losses (later based on RED)
- State variables:  
 $(W_n, S_n, M_n) := (CWND(t), SSTRESH(t), Loss(t))$
- Observation: TCP-Round



# A Single TCP Connection

---

**Input:** model state ( $W_n$ ,  $S_n$ ,  $M_n$ ), model factor ( $L(W_n)$ )

```
if ( $M_n = 0$ ) then {no loss last round}
   $S_{n+1} := S_n$ 
  if ( $W_n = W_{\max}$ ) then {full window possible}
     $W_{n+1} := W_n$ 
  else
    if ( $W_n < S_n$ ) then {slow start}
       $W_{n+1} := 2 * W_n$ 
    else {congestion avoidance}
       $W_{n+1} := W_n + 1$ 
    endif
  endif
else
```



# A Single TCP Connection

---

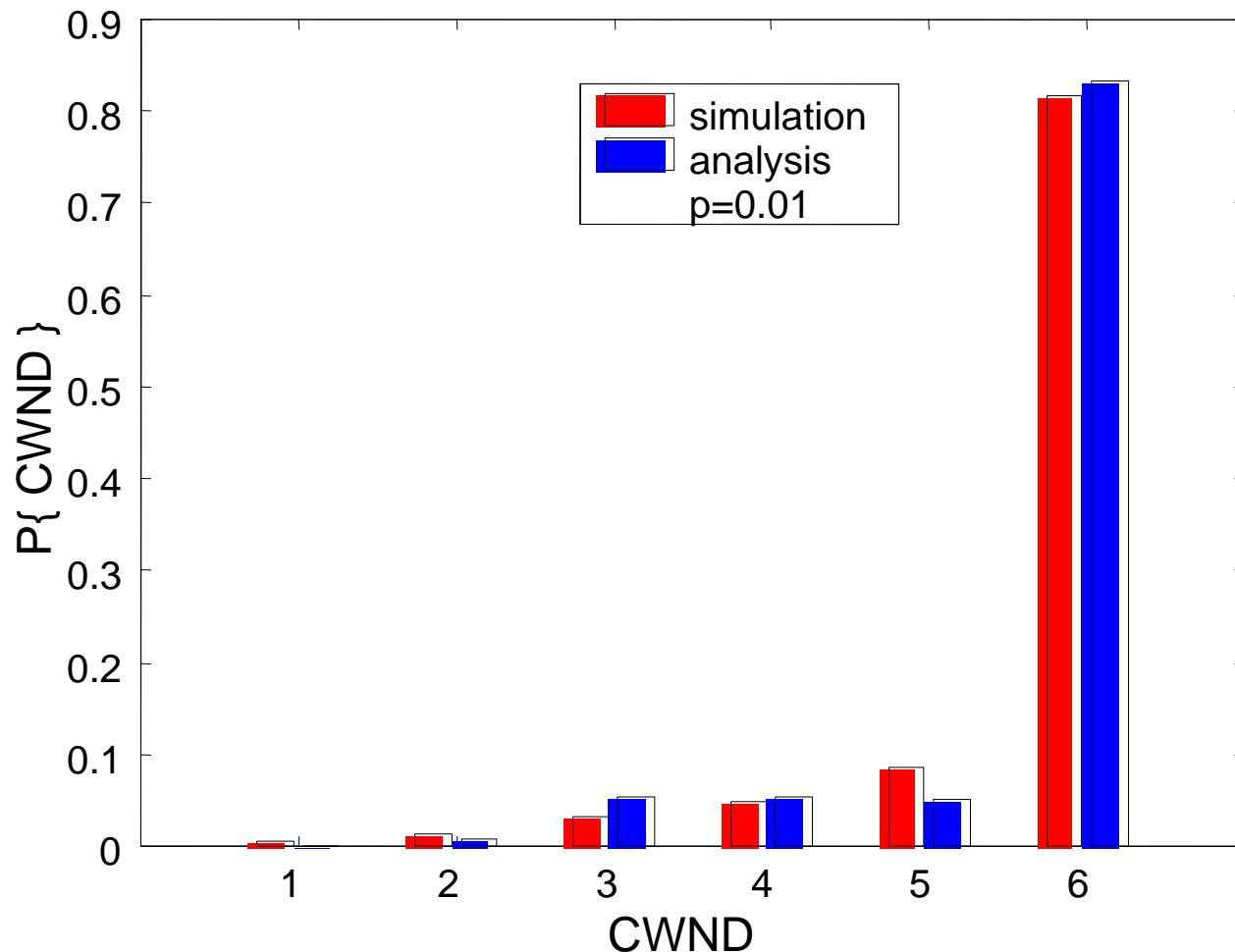
```
else
  if (Mn = 1) then          {one loss last round}
    Sn+1 := max(Wn/2, 2)
    Wn+1 := Sn+1
  else                      {more than one loss last round}
    Sn+1 := max(Wn/2, 2)
    Wn+1 := 1
  endif
endif
Mn+1 := min(L(Wn), 2)

Output: model state(Wn+1, Sn+1, Mn+1)
```



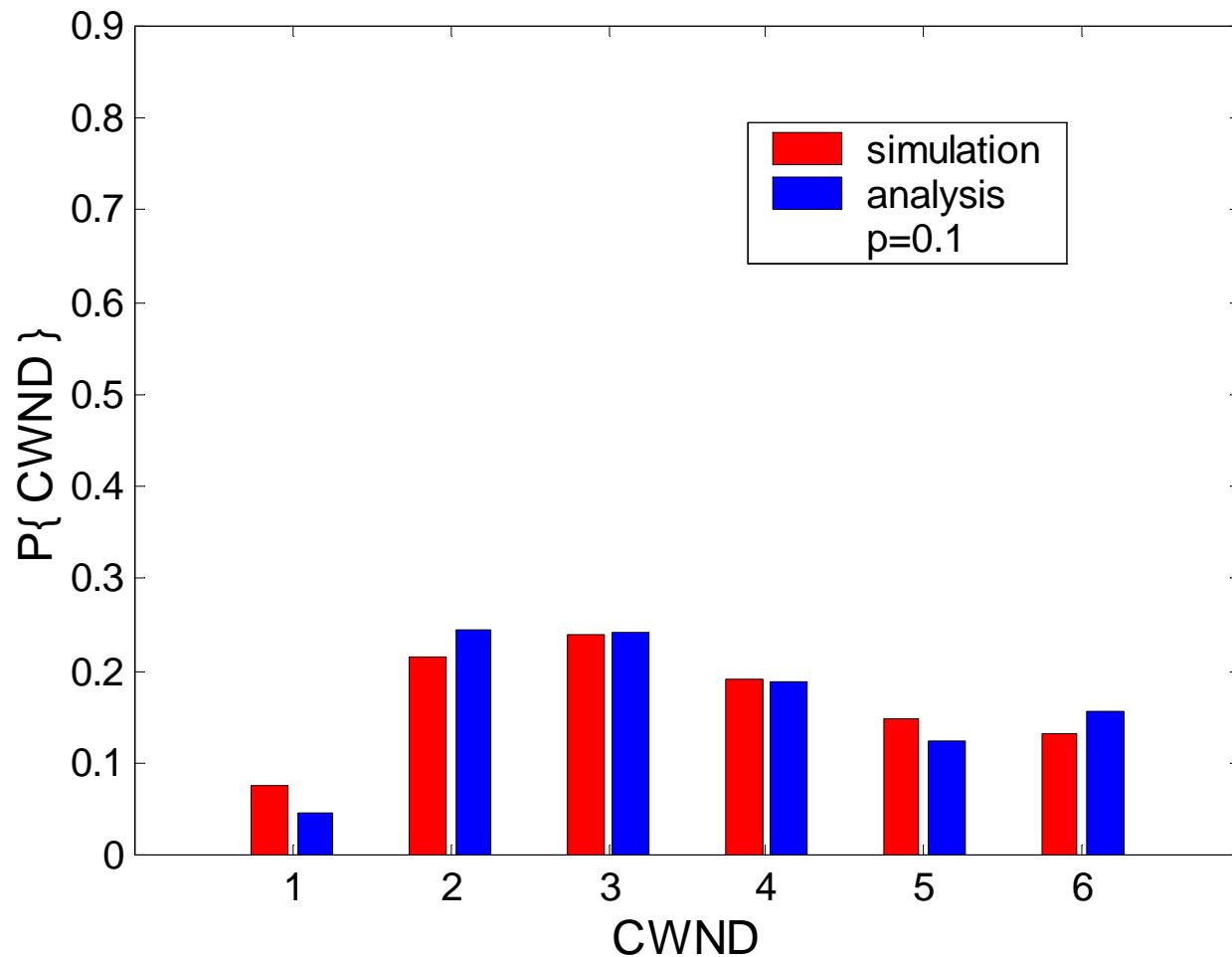
# Simulation $\leftrightarrow$ Analysis (I)

---



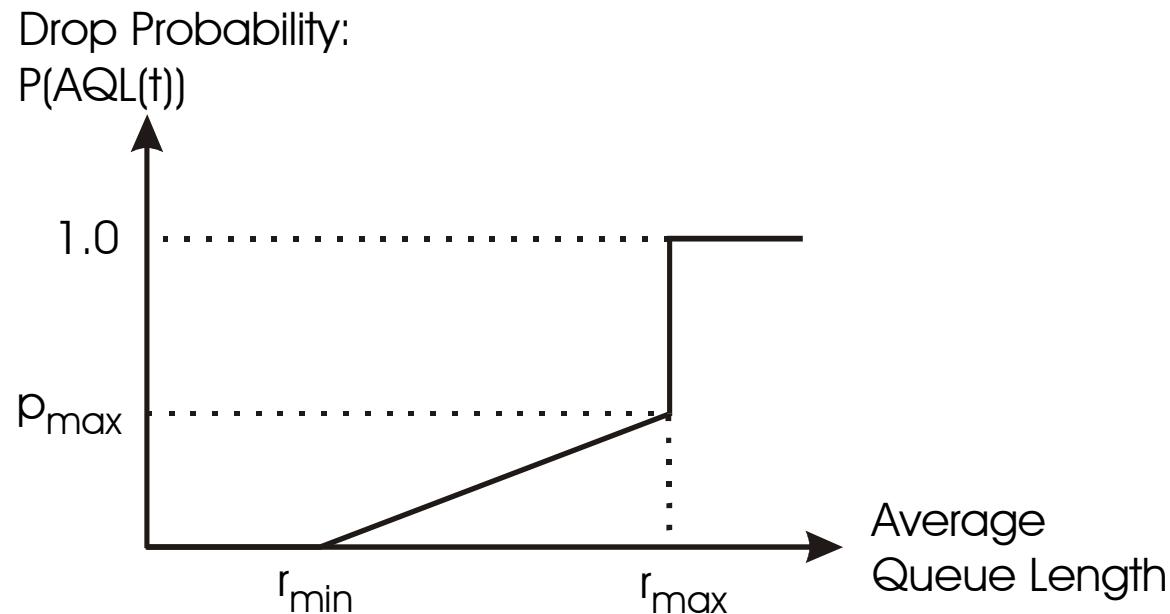
# Simulation $\leftrightarrow$ Analysis (II)

---



# Random Early Discard (RED) Queue

---



$$p(A=i) = \begin{cases} 0 & 0 \leq i < r_{\min} \\ \left( \frac{i - r_{\min}}{r_{\max} - r_{\min}} \right) \cdot p_{\max} & r_{\min} \leq i \leq r_{\max} \\ 1 & r_{\max} < i \leq \infty \end{cases}$$



# RED Queue Mechanism

---

**Input:** model state ( $A_n$ ), model factor ( $B, L(B, A_n)$ )

$$Q := B - L(B, A_n)$$

$$A_{n+1} := w_q \cdot Q + (1-w_q) \cdot A_n$$

**Output:** model state ( $A_{n+1}$ )

B: batch of packets

A: average queue size

L: loss

Q: actual queue size

$w_q$ : weighting factor

The probability of  $k$  losses within a batch  $B$  of  $j$  packets follows a binomial distribution:

$$l(A = i, B = j)[k] = \binom{j}{k} p(A = i)^k \cdot (1 - p(A = i))^{j-k}$$



# Compound Analysis

---

- ▷  $h$  TCP Connections under RED:

**Input:** model state  $((Wi_n, Si_n, Mi_n), (A_n))$ , model factor  $Li(A_n, Wi_n)$

```
for  $i \in \{1, \dots, h\}$  do  
   $(Wi_{n+1}, Si_{n+1}, Mi_{n+1}) := TCP((Wi_n, Si_n, Mi_n), Li(A_n, Wi_n))$   
end for
```

$$A_{n+1} := RED( (A_n), (S Wi_n, S Li(A_n, Wi)) )$$

**Output:** model state  $((Wi_{n+1}, Si_{n+1}, Mi_{n+1}), (Q_{n+1}, A_{n+1}))$





## Results

---

# Parameters

---

If not stated differently

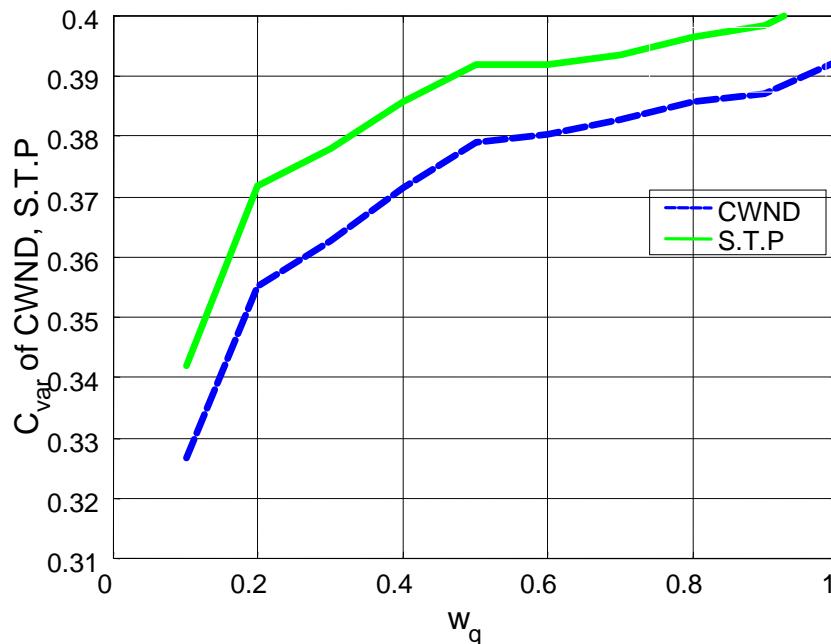
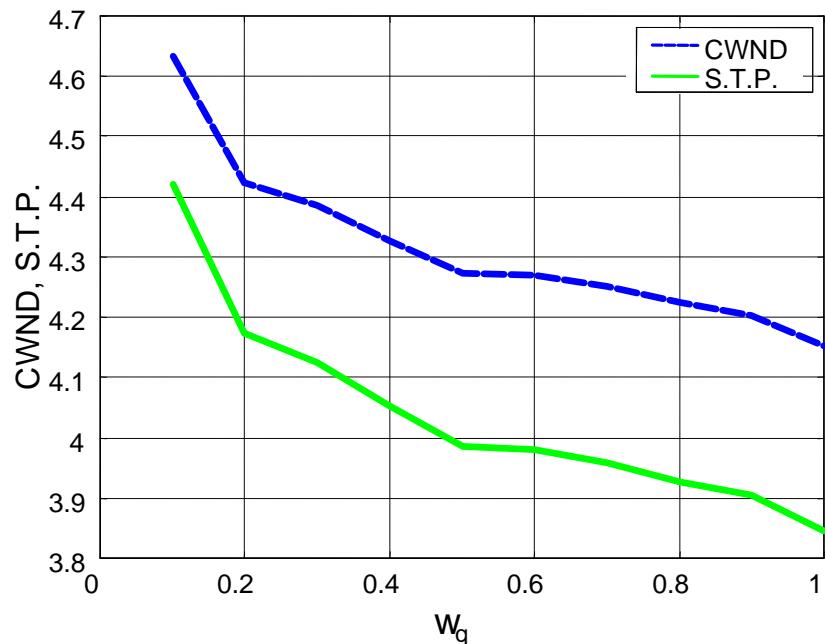
- $R_{\min}=9$
- $R_{\max}=18$
- $w_q=0.3$
- 3 TCP sources
- CWND=6



# Influence of the Weighting Factor

---

$$p_{\max} = 0.5$$

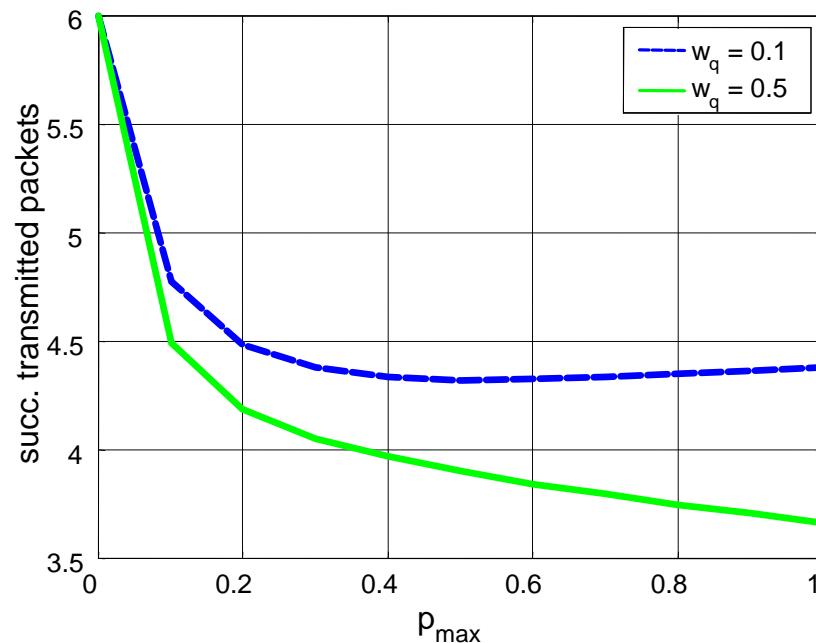
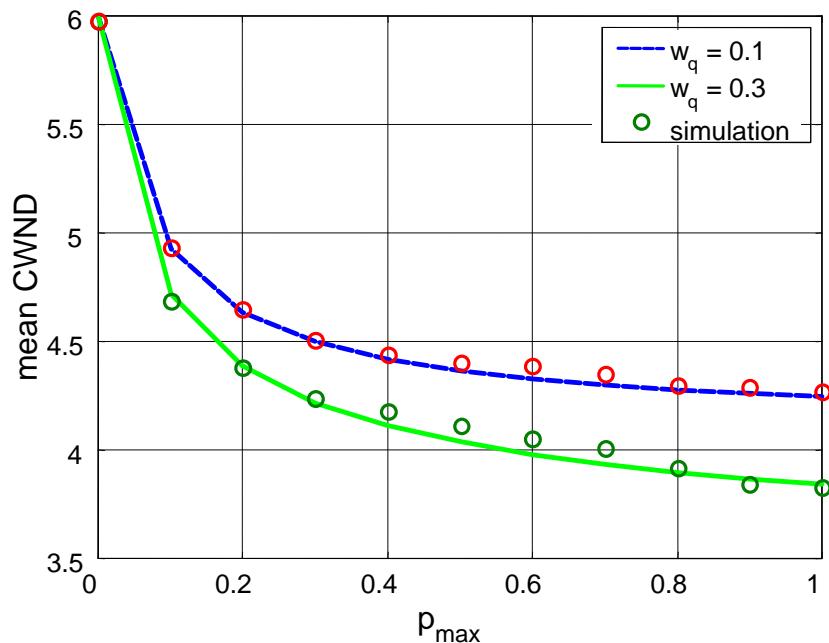


- ▷ Increased throughput, reduced variance



# Influence of the Loss Function

---

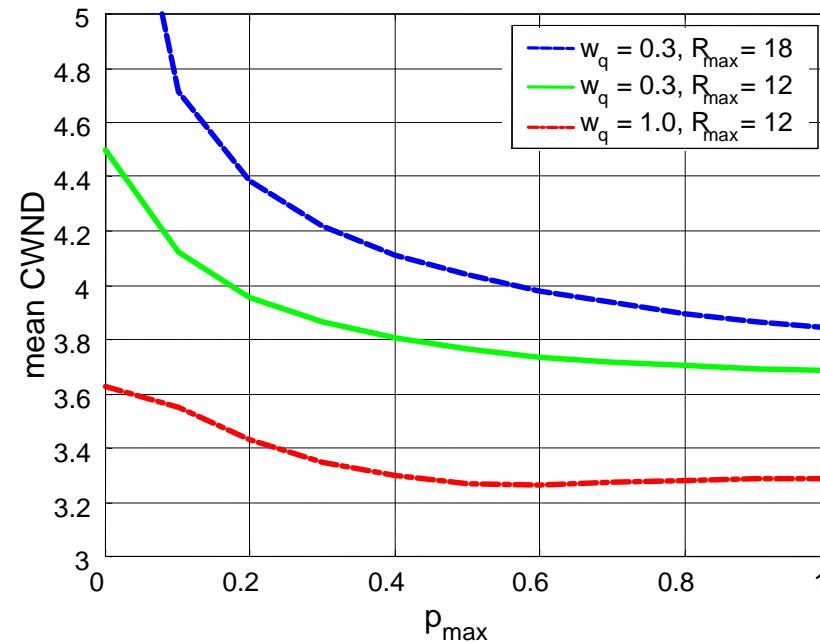
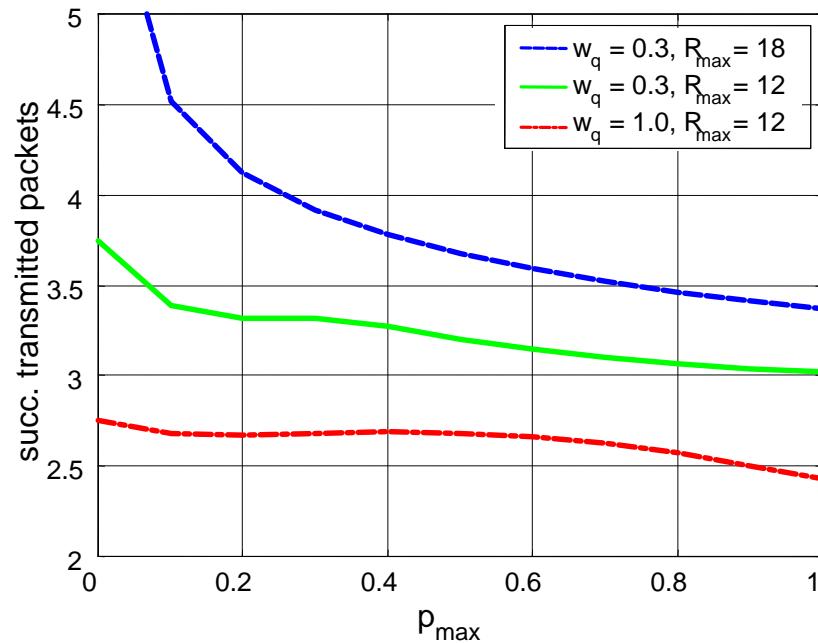


- ▷ High loss probability reduces throughput.
- ▷ “Long Memory” provides better results.



# Influence of the Buffer Size

$w_q = 0.3$

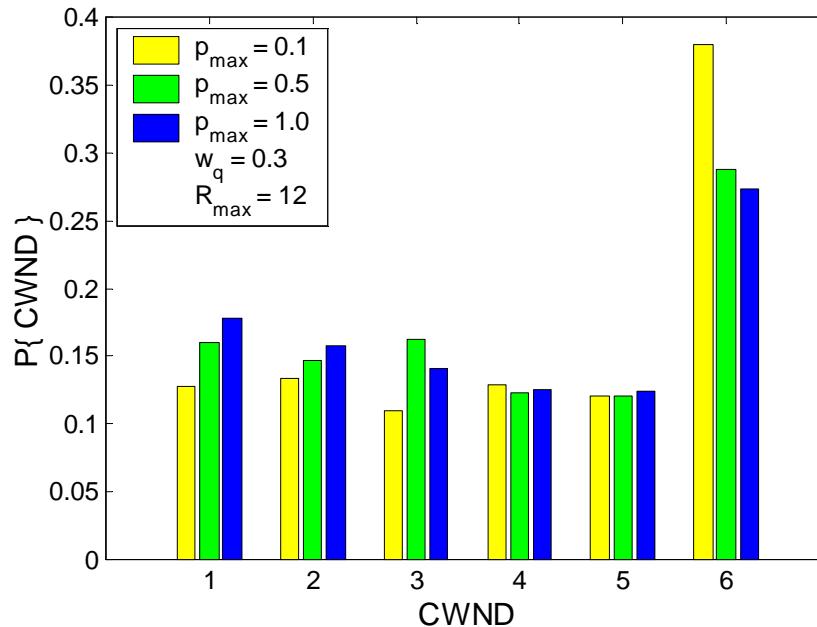
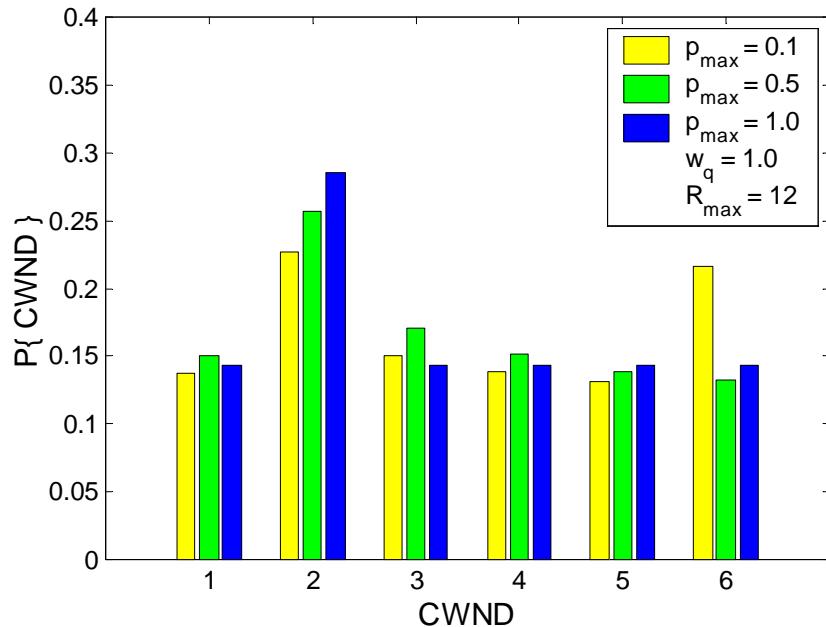


- ▷ Short (or congested) queues limit the congestion window size.
- ▷ FIFO  $\Leftrightarrow$  RED



# Distribution of CWND

---



- ▷ “Long Memory” and small loss probabilities show larger congestion windows



# Conclusion and Outlook

---

- ▷ Summary
  - Discrete-Time Model of TCP and RED
    - Correlation of TCP sources
  - Analytical Performance Evaluation
  - Distributions for all TCP state variables
- ▷ Results
  - Sensitivity of TCP to
    - Weighting factor
    - Loss function
    - Congestion or short queues
  - Good Performance: “Smoothed FIFO-Queues”
- ▷ Outlook
  - Comparison of different TCP implementations
  - Influence of non-linear loss functions
  - Fairness studies





---

# THE END

---