Multi-rate Queueing Loss and Processor Sharing Systems
A brief but incomplete Summary of recent Results

by Harro L. Hartmann

Institute for Computer Technology and Communication Networks
Technical University Braunschweig
E-mail: H.L.Hartmann@tu-bs.de

Abstract - Multi-rate systems fortunately operate in a complete sharing mode of high capacity links because this saves maximum load of each state by solving the stochastic knapsack problem at the burden of different (relaxed) service-specific performance values. 1981 J. S. Kaufman and J. W. Roberts published a multi-rate or multi-level recurrence equation which can be solved iteratively in polynomial time of $O(S\ C)$ in contrast to further approaches which require matrix evaluations or multidimensional convolutions. 1992 J.-F. P. Labourdette and G. W. Hart created an appealing approximation for the service type blockings by the assumption of an equivalent Erlang system for integer states near C and complexity $O(1)$. 1994 D. Mitra and J. A. Morrinson proposed the direct and high accurate approximation of service blockings in the transform domain by a contour-integral technique for states n near C and complexity $O(1)$. These contributions were devoted to buffer-less systems and integer service type bandwidths only.

In 2000 the author discovered a one-level functional equation the step parameter of which can be determined by solving a constrained identity requirement for the multi- and one-level solution in the transform domain. The resulting one-level step parameter then proves to be the effective multi-rate recurrence depth d which state-dependently varies between the maximum and minimum bit rates of the source ensemble. Then $N = C/d$ indicate the distinguishable number of multi-rate connections under progress. A rough state-independent estimate of d is the expected value of the offered traffic bit-rates and this is verified to be a key parameter for further investigations.

Thus, this contribution proceeds towards queueing loss and processor sharing (PS) systems. A huge amount of analytical difficulties remains prevented by a carefully adjusted enhancement of model precisions. First, the generalized recurrence equation of multi-level M/M/C/C_b feed-forward queueing loss system is iteratively solved where C_b denotes the possible finite buffer capacity. It may be converted to a state-equivalent one-level model too, which provides a near explicit solution. This one-level model again establishes down-compatibility to traditionally known queueing systems including generally distributed service times. Finally M/G/C systems with partial and strict processor sharing service disciplines are compared. All provided one-level solutions uniquely use the incomplete Gamma function and accept continuous service-type bit-rates. The state probability densities are obtained with time-complexity $O(1)$ per state and the approximation errors remain sufficiently small in comparison to unavoidable demand forecast errors at any planning horizon.

Keywords - Multi-rate queueing loss systems, multi-rate processor sharing systems.
Further material - The author's homepage http//:www.tu-bs.de/~hlhartma