Mathematical Modeling of Self-Organizing Systems

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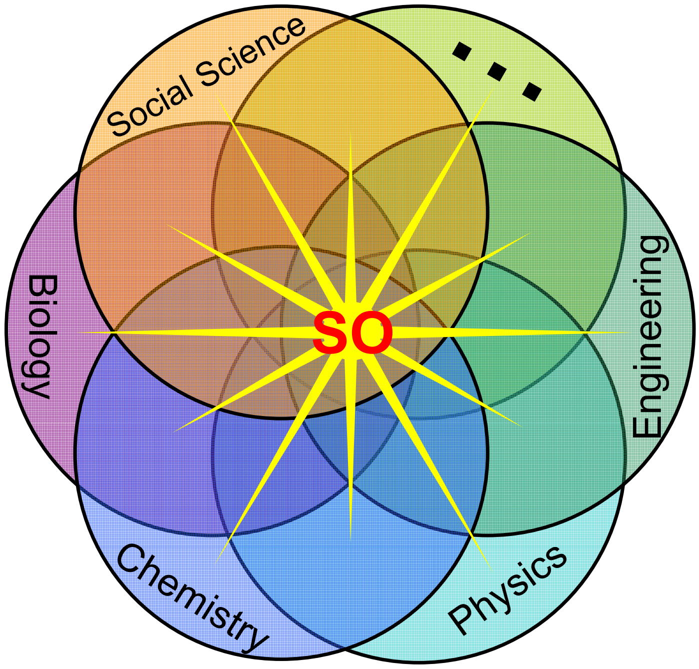
Overview

1. Examples of Self-Organizing Systems
2. The Need for Modeling Techniques
3. Classification of Self-Organizing Systems
4. Modeling of Self-Organizing Systems
5. Future Project Directions
6. Conclusions

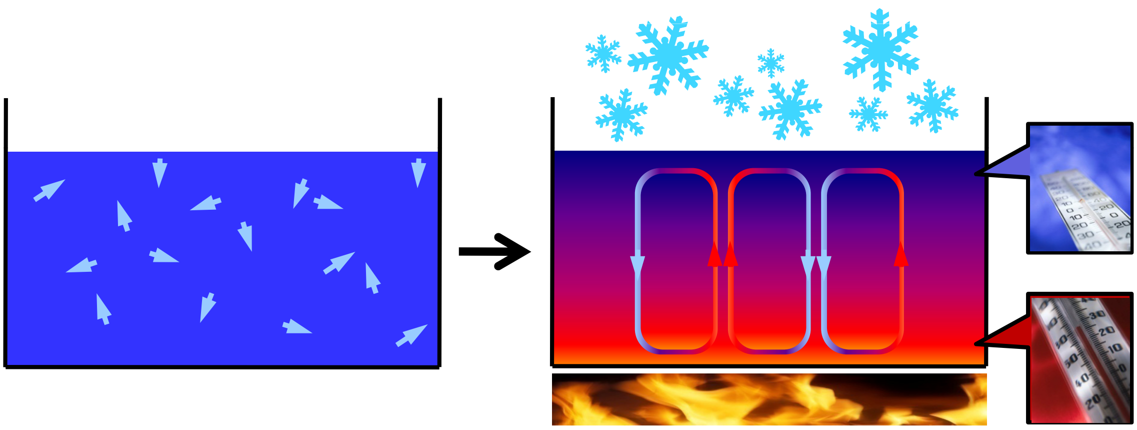
1. Examples of Self-Org. Systems

- Occurrence of Self-Organizing Systems



1. Examples of Self-Org. Systems

- Physics Example: Benard Cells [1]
  - Molecules self-organize in heated liquid.
  - No external entity imposing rules.



1. Examples of Self-Org. Systems

- Biology Examples
  - Ant colonies marking paths using pheromone
  - Fireflies
  - Human brain
  - "Game of Life"
- Chemistry Examples
  - Nonlinear chemical oscillators, e.g., Belousov-Zhabotinsky reaction [1,2,3]
- Engineering Examples
  - Intended
    - Bio-inspired protocols, e.g., ant routing [4]
    - Intrusion detection using self-organizing maps
    - Increasing the degree of self-organization in P2P systems [5]
  - Unintended
    - Self-similiarity of Internet throughput
    - Self-synchronization among internet routers

2. The Need for Modeling Techniques

- Understanding self-organization is extremely important for understanding the behavior of today's and future complex systems.
- How can we understand complex systems?
- The modeling of systems makes a fair contribution to the understanding of these.

➔ Building abstract models of complex, self-organizing systems.

3. Classification of Self-Org. Systems

- Definitions of self-organization and self-organizing systems
  - “The evolution of a system into an organized form in the absence of external pressures.” [6]
  - “A move from a large region of state space to a persistent smaller one, under the control of the system itself. This smaller region of state space is called an attractor.” [6]
  - “The introduction of correlations (pattern) over time or space for previously independent variables operating under local rules.” [6]
  - “The spontaneous emergence of global coherence out of local interactions.” [7]
  - “Self-organization is a process where the organization (constraint, redundancy) of a system spontaneously increases, i.e. without this increase being controlled by the environment or an encompassing or otherwise external system.” [8]
  - “The appearance of structure or pattern without an external agent imposing it.” [7]

3. Classification of Self-Org. Systems

- Rather necessary properties of self-organizing systems
  - “[The] **increase of coherence**, or decrease of statistical entropy, [...] defines self-organization.” [7]
  - **Global order:** emerges from local interactions. [4,6]

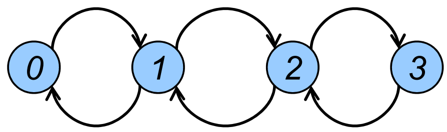
3. Classification of Self-Org. Systems

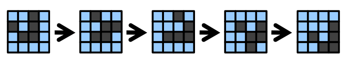
- Rather optional properties of self-organizing systems
  - **Autonomy:** absence of external control (≠autarchy).
  - **Dynamic operation:** time evolution.
  - **Dissipation:** system consumes energy/matter/information taken from environment.
  - **Instability:** nonlinearity caused by feedback .
  - **Redundancy:** multiple components of similar type, insensitivity to damage.
  - **Self-maintenance:** repair or recreate defect components.
  - **Adaptation:** compensating external perturbations.
  - **Complexity:** difficult to describe the semantics overall system behavior.
  - **Hierarchies:** multiple nested self-organized levels.
  - **Fluctuations:** searching through options through noise.
  - **Symmetry breaking:** small fluctuations → branch selection at bifurcation/critical points.
  - **Multiple equilibria:** many possible attractors.
  - **Criticality:** threshold effects, chain reactions possible.

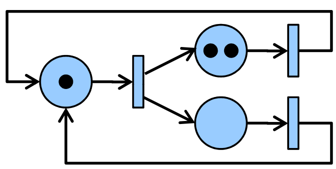
[4,6]

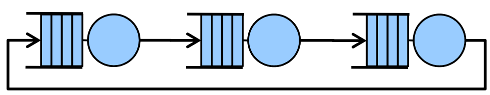
4. Modeling Self-Org. Systems

- Well-known modeling techniques
  - Markov chains [9]
    - Discrete-time/continuous-time Markov chains
    - Embedded Markov chains
    - Structured Markov chains
    - Reward models
  - Cellular automata [10]
    - Synchronous updating
    - Asynchronous updating
  - Stochastic Petri nets [9]
    - Generalized stochastic Petri nets
    - Deterministic and stochastic Petri nets
    - Extended stochastic Petri nets
    - Colored Petri nets
    - Hierarchical Petri nets
  - Queueing networks [9]
    - Product-form queueing networks
  - Network calculus [11]
  - Simulation models [9]
  - Stochastic process algebras [12]
  - Fault trees, reliability block diagrams, reliability graphs, task graphs, … [12]
  - Hybrid approaches









4. Modeling Self-Org. Systems

- Specific modeling approaches used for self-organizing systems
  - Markov chains
    - Social network formation [13]
  - Network calculus
    - Sensor networks [14]
  - Simulation models
    - Too numerous to count
    - Example applications
      - Social networks
      - Sensor networks
      - Neural networks
      - Mesh transport networks
      - Neuro-mechanical networks
      - Load balancing in grids

5. Future Project Direction

- Focusing on a general modeling framework and evaluation methodology
- Solving application specific problem using the new framework
- Providing tool support for the convenient usage of the developed methodologies

6. Conclusion

- Self-organization plays an important role in current and future communication systems.
- Self-organization is a concept that is not only studied in engineering disciplines.
- The artificial introduction of self-organizing behavior can lead towards positive and negative effects.
- However, large complex dynamic systems always tend to self-organize – if intended or not.
- The modeling and evaluation of self-organizing behavior is of utmost importance.
- A general mathematical modeling framework, applicable in many application areas, seems helpful but does not yet exist.

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