A New Peer-to-peer Network

Hovhannes Harutyunyan and Junlei He
Department of Computer Science and Software Engineering
Concordia University, Montreal, QC, Canada
E-mail: haruty@cs.concordia.ca

Outline

- Introduction
- The Knödel Graphs
- P2P Networks Based on Knödel Graphs
- Conclusions and Future Work

Introduction – Goal of Study

- Improve routing efficiency of P2P network
  - Find shorter routing path
  - Decrease size of routing table on each node

Introduction – Related work

- Gnutella
  - Decentralized
  - Non-deterministic
  - Efficiency problem
- CHORD
  - Deterministic
  - Routing length: $O(\log N)$
- CAN
  - Average routing length: $(d/4)(N^{1/d})$

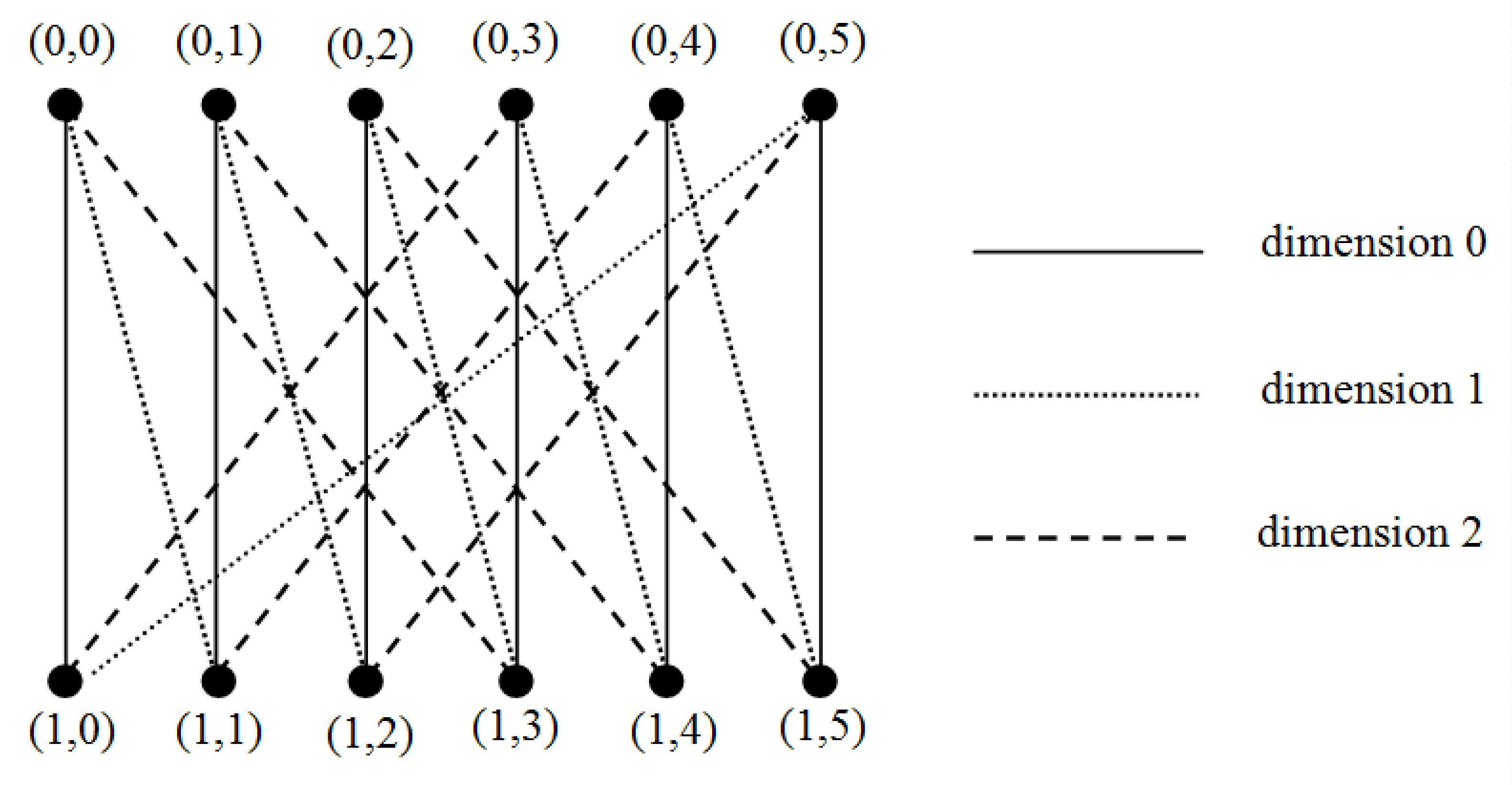
Knödel Graphs – Definition

The Knödel graph on $n \geq 2$ vertices (n even) and of maximum degree $d, 1 \leq d \leq \lfloor \log_2 n \rfloor$ is denoted by $W_{d,n}$.

The vertices of $W_{d,n}$ are the pairs (i, j) with $i = 0,1$ and $0 \leq j \leq \frac{n}{2} - 1$, and the set of edges:

$$E = \left\{ \left((0, i), (1, j) \right) \mid j = i + 2^r - 1 \bmod \frac{n}{2}, 0 \leq i, j \leq \frac{n}{2} - 1,\ 0 \leq r \leq d - 1 \right\}$$

Knödel Graphs – Sample $W_{3,12}$



Partial Knödel Graphs

- A full Knödel graph has dimension d

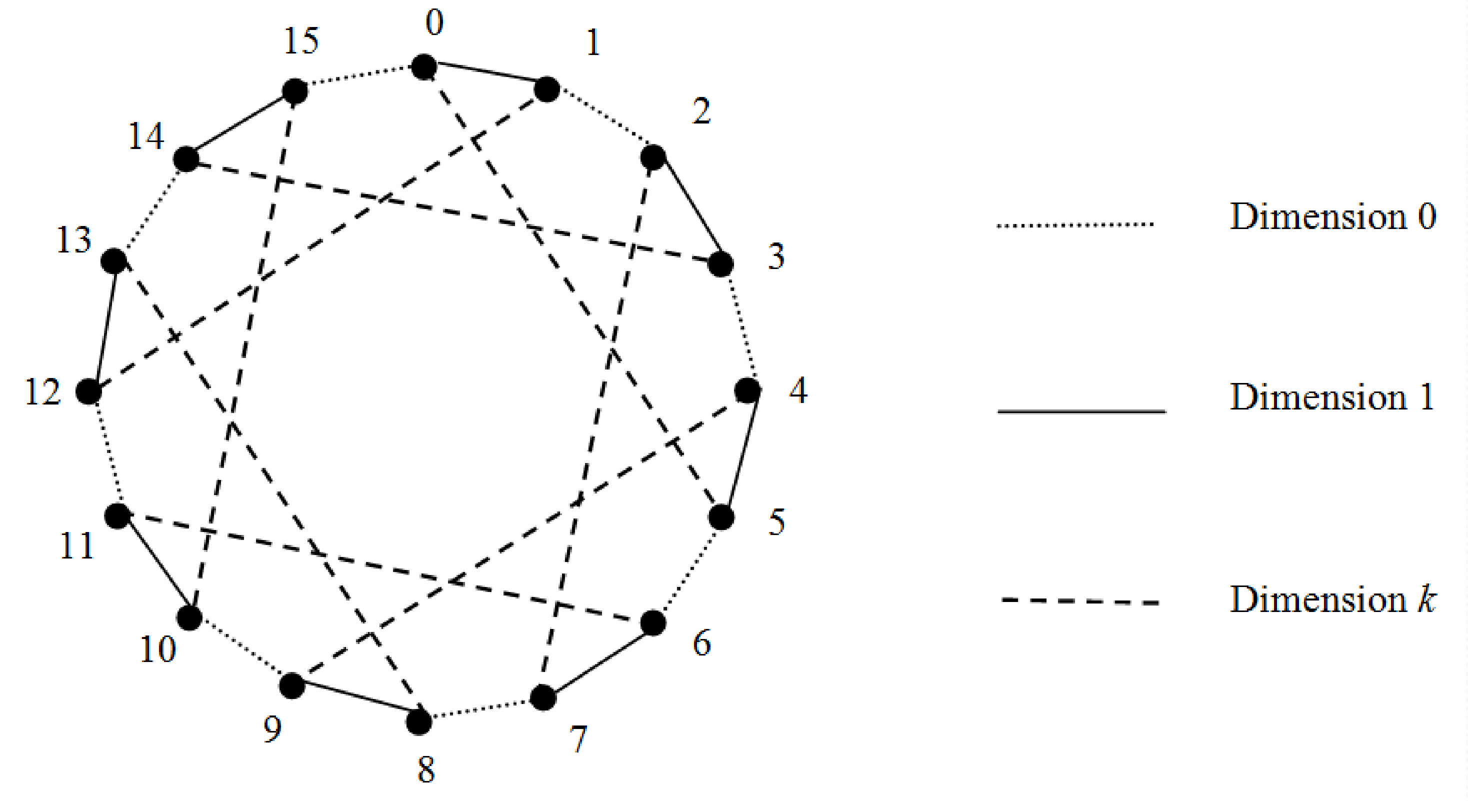
- Partial Knödel graph has dimension less than d: dimension 0, 1, and some dimensions between 2 and d-1

- Renumber nodes in $W_{d,2^d}$ to simplify discussion:

$$\begin{cases} (0,i) \Rightarrow 2i \\ (1,j) \Rightarrow (2j-1) \bmod 2^d \end{cases}$$

Partial Knödel Graphs – Sample

- Partial Knödel graph $W_{4,16}$ of 3 dimensions, k=2



Routing in Partial Knödel Graph

- Determine in which half the destination node is located.
- Determine the distance between the destination and current position along the cycle.
- Use the edges of the appropriate dimension and dimension 0 or 1 to approach the destination.
- Use dimension 0 and 1 alternatively to reach the destination along the Hamiltonian cycle.

Routing Length: $O((i+1)*\sqrt[i+1]{N})$ (i is the number of degrees)

Routing in Full Knödel Graph

- Consider routing from node 0 to node *x*

- The binary representation of integer *x* shows a path from node 0 to node *x*

- For even x: $x = \sum_{t=1}^{m} (2^{i_t} - 2^{j_t}) \quad d \geq i_1 > j_1 > i_2 > j_2 > \cdots > i_m > j_m > 0$

- For odd x: $x = \sum_{t=1}^{m} (2^{i_t} - 2^{j_t}) + 2^s - 3, (s \in \{1,2,3\})$

Routing in Full Knödel Graph

- Example: d=10, x=414

$$x = 414 = (0110011110)_2 = (2^{8+1} - 2^{6+1}) + (2^{4+1} - 2^{0+1})$$

- This shows a path from node 0 to node 414: dimension 8, dimension 6, dimension 4, and dimension 0 (node 0 – node 509 – node 384 – node 413 – node 414). The length of this path is 4.

Routing in Full Knödel Graph

- Reduction rules
  - Rule1:

$$2^a + 2^b - 2^{a-1} - 2^{b+1} = 2^{a-1} - 2^b$$

  - Rule2:

$$2^a + 2^b + 2^c - 2^{a-1} - 2^{b-1} - 2^{c+2} = 2^{a-1} + 2^{b-1} - 2^{c+1} - 2^c$$

  - Rule3:

$$2^a + 2^b + 2^c - 2^{a+1} - 2^{b+1} - 2^{c-2} = 2^{c-1} + 2^{c-2} - 2^a - 2^b$$

  - Rule4:

$$2^d + 2^a - 2^{a-1} (\mathrm{mod}\ 2^d) = 2^{a-1}$$

Routing in Full Knödel Graph

- By applying the reduction rules, the routing length in Knödel graph may be reduced to $\left\lceil \frac{d+2}{2} \right\rceil$

P2P Network Based on Knödel Graph – Basic Design

- Associating each data item with a key, and hashing the keys to each node of the Knödel graph.
- Each peer is in charge of an arc of the Hamiltonian cycle of the Knödel graph.
- Each peer maintains a routing table which contains *d* items – the edges in the Knödel graph.
- When searching for certain content, first find the node holding the key, and then find the peer in charge of the node, forward the querying message to the peer, and retrieve the address of the peer which owns the content.

P2P Network Based on Knödel Graph – Routing Partial Knödel Graph

```
peer::findKey (key)
{
    if( current peer is in charge of key )
        return this;
    peer1 = closest available successor to key in routing table;
    if( peer1 = peer )
    peer1 = the furthest peer available in routing table;
    return peer1.findKey(key);
}
```

P2P Network Based on Knödel Graph – Routing Full Knödel Graph

```
peer::findKey (key) {
    if( current peer is in charge of key )   return this;
    calculate the binary representation;
    dist = key – this->keyValue;
    peer1 = closest peer in the binary representation to dist;
    if( peer1 = peer )
        peer1 = the furthest peer available in routing table;
    return peer1.findKey(key);
}
```

P2P Network Based on Knödel Graph – Efficiency Analysis

- Length of routing path
  - P2P network based on partial Knödel graph: $O((i+1)\sqrt[i+1]{N})$
  - P2P network based on full Knödel graph: $O(\log N)$

- Size of routing table on each node
  - P2P network based on partial Knödel graph: i (not related to N)
  - P2P network based on full Knödel graph: $O(\log N)$
  - Practically the size is much smaller (see simulation result)

Simulation Results

- For the Knödel graph $W_{d,2^d}$, simply applying the reduction rules in the same order may reduce the average routing length by $1/3$ ($d/2$ before reduction, $d/3$ after reduction), and the maximum routing length is about $2d/3$.
- The average and max routing length in the P2P network based on Knödel graph is smaller than that in CHORD and CAN of the similar network size, while the size of routing table is also smaller.

Conclusion & Future Work

- The P2P network based on the Knödel graph improved the routing efficiency.
- The P2P network based on the partial Knödel graph may be used in mobile environment where computing resources are limited.
- We will further improve the routing algorithm for the Knödel graph to make the routing length even shorter.

Thank you!