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A Memory Markov Chain Model For VBR Traffic With Strong Positive Correlations

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Abstract
We propose a new modeling approach for variable bit rate (VBR) traffic in packet networks based on a Markov chain with memory. The model is simple, comprehensive, and facilitates the modeling of strong positive correlations over a considerable range of lags. The model can easily be adapted to measured traffic sequences, e.g. MPEG video or packet interarrival time sequences. Due to its simplicity and its close relationship to Markov chains the model is valuable to both simulation and analysis of currently considered types of traffic.

1 Introduction

Currently, the increasing amount of traffic types with strong positive correlations or even long-range dependent behavior in combination with larger buffers in the telecommunication equipment demand for traffic models and analytical tools capable to cope with this new scenario. For instance, if real-time VBR video traffic is considered as being uncorrelated, performance analysis will underestimate losses and delays at network buffers by orders of magnitude [6, 5].

Thus, there is a demand for models facilitating the modeling of such correlation characteristics. In particular, the modeling of VBR video traffic attracted a lot of interest, e.g. see overviews [2, 7]. Among those modeling approaches, there are Markov chains, autoregressive processes, TES models, selfsimilar processes, and even combinations of these. The simple modeling approaches, such as histograms or first-order Markov chains, are not capable to provide a good approximation of the autocorrelation function of the empirical data sets. The advanced modeling techniques, such as selfsimilar processes, lead to difficulties in parameter estimation, simulation, and analysis.

We therefore propose a new approach for modeling VBR traffic: the Memory Markov Chain (MMC). The main idea is to redefine the set of states of a first-order Markov chain. Instead of representing only the sample size, as for a conventional Markov chain, in our model a state represents both the sample size and the average of the sizes of a number of preceding samples. This approach is different from a higher-order Markov chain, since the MMC does not take into account the the preceding states but their averaged effect. Thus, we avoid a state space explosion which is typical for higher-order Markov chains. Nevertheless, we are able to improve the correlation behavior considerably.

The parameter estimation of the MMC requires three parameters: the number of states M_S representing the sample size, the number of samples W which are considered for the averaging process, and the number of states M_A representing the average sizes. Having defined the MMC states, the generation of the transition probability matrix is as straightforward as for a first-order Markov chain.

In short, the Memory Markov Chain is a modified first-order Markov chain with improved correlation properties. It can be applied in analysis and simulation of buffers of considerable size.

In the following, we line out the reasoning behind our approach and the parameter estimation. We apply the MMC approach to the modeling of MPEG video traffic and show the validity and usefulness of the model by presenting autocorrelation functions and cell loss estimates of an ATM buffer with MPEG video input.

2 The Memory Markov Chain and its parameter estimation

Let $\{x_i : i = 1, \ldots, N\}$ denote the considered traffic measurement time series and let W denote the number of samples considered for the averaging process. The given trace $\{x_i\}$ is now extended to a series of pairs $\{(x_i, \bar{x}_i)\}$ with

$$\bar{x}_i \quad = \quad \frac{1}{W} \sum_{k=i-W-1}^{i-1} x_k \quad \text{for} \quad i = W+1, \ldots, N. \tag{1}$$

The series of pairs consists of W samples less than the original time series.

The averaging process is motivated as follows. For a time series with fast decaying autocorrelation function, the values $\bar{x}_i$ will be rather good estimators of the sample mean and will be almost the same for all values of i. Hence, this additional category contains no information and is of very limited value with respect to a better characterization of the sample correlations. In the presence of a slowly decaying autocorrelation function, however, the values $\bar{x}_i$ differ significantly and represent the cumulated averaged history of x_i.

Before we start to determine the transition matrix for the MMC we have to define an appropriate state space. Each pair of the series $\{(x_i, \bar{x}_i)\}$ is related to a state (m_i^S, m_i^A), where $m_i^S \in \{1, \ldots, M_S\}$ denotes the sample size class and $m_i^A \in \{1, \ldots, M_A\}$ the average size class of sample i. The classes are obtained by discretizing both the x_i and $\bar{x}_i$ as follows.

$$m_i^S \quad = \quad \begin{cases} 1 & \text{if} \quad x_i \quad = \quad \min_i x_i \\ \left\lceil \dfrac{x_i - \min_i x_i}{\max_i x_i - \min_i x_i} \cdot M_S \right\rceil & \text{otherwise} \end{cases} \tag{2}$$

The $\bar{x}_i$ are processed analogously.

The $M_S \cdot M_A$ states are ordered such that the states for one average size class are grouped together with ascending average size class number. The entries of the transition matrix

$\mathbf{P}$ can now be estimated as usual as empirical transition probabilities [1]. The size vector $\mathbf{s}$ related to the states of the Markov chain is determined in two steps. First, we compute the size vector of length M_S based on the x_i values.

Let $\delta(m_i^S, j)$ denote the indicator function of $m_i^S = j$ with $j \in \{1, \ldots, M_S\}$. The sizes s_j related to sample size class j are now given by

$$s_j = \frac{\sum_{i=W+1}^{N} \delta(m_i^S, j) \cdot x_i}{\sum_{i=W+1}^{N} \delta(m_i^S, j)}. \tag{3}$$

Then, vector $\mathbf{s}$ is formed by concatenating M_A copies of that vector. For instance, let $M_S = 3$ and $M_A = 2$. Then, the state space consists of $\{(1,1), (2,1), (3,1), (1,2), (2,2), (3,2)\}$ and $\mathbf{s} = [s_1, s_2, s_3, s_1, s_2, s_3]$.

The MMC model is fully characterized by the transition matrix $\mathbf{P}$ and the size vector $\mathbf{s}$. The mean and variance of the process generated by this model are given by

$$\mu = \boldsymbol{\pi} \cdot \mathbf{s}^T, \qquad \sigma^2 = \sum \boldsymbol{\pi} \cdot diag(\mathbf{s})^2 - \mu^2 \tag{4}$$

where the vector $\boldsymbol{\pi}$ is the steady-state distribution of the MMC given by the solution of

$$\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P}, \qquad \sum \boldsymbol{\pi} = 1. \tag{5}$$

and $diag(\mathbf{s})$ denotes a square diagonal matrix with diagonal $\mathbf{s}$. The correlation coefficient for lag k is obtained by

$$\rho_k = \frac{1}{\sigma^2} \left[\sum \boldsymbol{\pi} \cdot diag(\mathbf{s}) \cdot \mathbf{P}^k \cdot diag(\mathbf{s}) - \mu^2 \right]. \tag{6}$$

Note, that a sequence generated by means of transition matrix $\mathbf{P}$ and the size vector $\mathbf{s}$ will have a marginal distribution which is identical to a histogram with M_S intervals of equal size.

The values M_A and W have to be determined empirically. Experiments have shown that that M_A should be at least 5 to clearly outperform a standard first-order Markov chain. If the MMC should be capable to approximate the first L lags then the parameter W should be chosen in the neighborhood of L. There are values of M_A and W where an increase of these values leads only to marginal improvements of the MMC correlation properties. This is due to the usage of finite traces for the estimation of the transition matrix leading to a loss in statistical significance of its entries for increasing values of M_A. The parameter M_A should therefore be chosen as small as tolerable in terms of model accuracy.

3 MPEG video traffic example

To show the usefulness and the validity of our approach, we applied the MMC as a model for variable bit rate MPEG video. As data sets, we chose the *dino* (Jurassic Park) sequence from the University of Würzburg data sets [4] and the Bellcore MPEG *Star Wars* sequence [3]. Details on MPEG video and statistical properties of the video data sets can be found in the related literature. In the sequel, we will only show the results for the *dino* sequence. The results for the *Star Wars* sequence are essentially the same.

Figure 1 shows the autocorrelation functions for a number of MMC models fitted to the *dino* GOP size time series where Group Of Pictures (GOP) is a number of consecutive MPEG video frames. We did not use the frame size sequence since in this case the strong positive correlations are hidden due to the periodic structure of the frames sizes. For details on the correlation structure of MPEG video see [4].

All MMC models have $M_S = 20$ GOP size classes and $M_A = 10$ average GOP size classes, implying a 200×200 transition matrix. The parameter W was varied from 1 to 100. Note, that a value of $W = 1$ results in the autocorrelation curve of the simple Markov chain model. As expected, the models' capability in approximating the empirical autocorrelation function increases with larger values of W. In addition, the substantial difference of the correlation characteristics of a conventional first-order Markov chain and the MMC with a memory of tens of past samples becomes obvious.

As an additional means of validation, we applied the MMC model of VBR MPEG video traffic to predict cell losses at an ATM multiplexer buffer with *dino* input traffic. Again, we use the MMC approach to model the GOP size sequence. The frame sizes are computed from the GOP sizes by multiplying constant factors representing the average percentage of the individual frame sizes of a GOP compared to the average GOP size (for details see [6]).

Figure 2 shows the cell loss estimates at an ATM buffer with 90% load for an uncorrelated histogram model with 20 intervals, a conventional first-order Markov chain with 20 states and an MMC with parameters $M_S = 20$, $M_A = 10$, and $W = 100$ compared to the losses of the empirical data set. Thus, the marginal distributions of all generated sample sequences are the same. The models differ only in their capabilities to approximate the autocorrelation properties of the empirical data set and, of course, in complexity. For

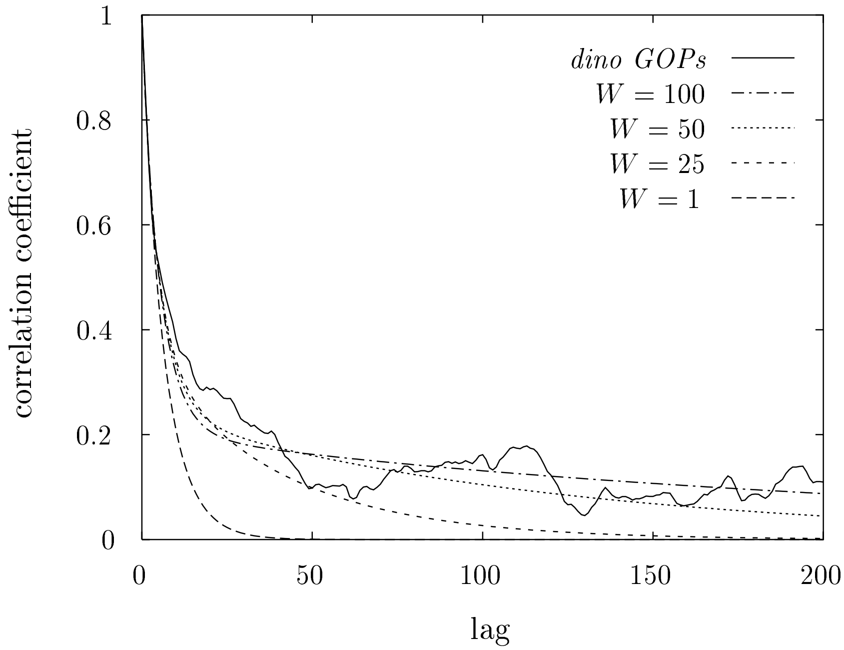

Figure 1: Autocorrelation functions of the Memory Markov Chain model

small buffer sizes, all modeling approaches lead to a very good estimate of the cell losses of the empirical data set. Increasing the buffer size, leads to very optimistic estimates for the histogram model. Increasing the buffer size further, the estimates of the conventional Markov chain become worse. Only the MMC shows the capability to estimate the cell losses for the whole range of buffer sizes with high accuracy.

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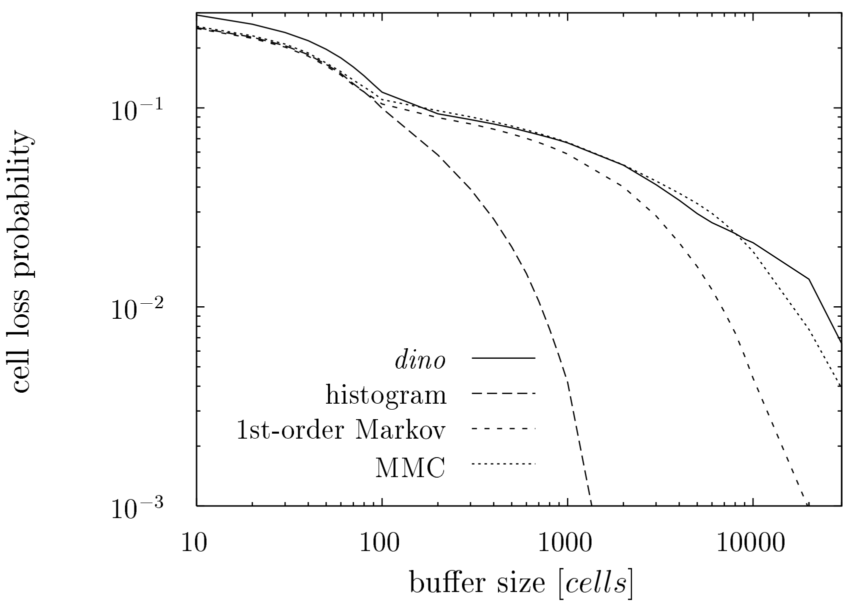

Figure 2: Cell losses ($\rho = 0.9$)

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